Invariant numerical PI control of analog processes

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The article presents Proportional Integral (PI)regulation of a production object with an analog measure of the final (output) output using a regulator whose control function is programmed, calculated and affects the object in a discrete (numerical) mode. The article describes the characteristics of the object and regulator, the conditions for tuning the regulator under which the "object-regulator" system will operate smoothly and stably, and the limits under which the system will be invariant. To compile the text, the author is used his own work "Theory of Invariance", informational materials of the Siemens company about the controllers from the Simatic and Logo systems, as well as data from the Wikipedia internet encyclopedia.

The need for information about the features of numerical programming necessitates adding to the old concepts characterizing PI regulation additional concepts from the theory of numbers for setting up a PI regulator programmed in a controller. The author uses modern algebra to clarify the criteria for tuning the proportional part of a numerical PI controller. Thus, the concept of "invariant control" and the methods for its application are clarified in a strictly mathematical manner.

***Keywords** – automatic control, invariance, numeric regulator.*

Инвариантно числено ПИ регулиране на аналогови процеси (Милан М. Станков). *Статията представя Пропорционално и Интегрално (ПИ) регулиране на производствен обект с аналогова мяра на крайната (изходната) продукция с помощта на регулатор, чиято управляваща функция се програмира, изчислява и въздейства на обекта в дискретен (числен) режим. Статията описва характеристиките на обекта и регулатора, условията за настройване на регулатора, при които системата „обект – регулатор" ще работи плавно и устойчиво, както и границите, при които системата ще бъде инвариантна. За съставянето на текста авторът е ползвал собствения си труд „Теория на инвариантността", информационни материали на фирмата Siemens за контролерите от системата Simatic и Logo, както и данни от енциклопедията в интернет Wikipedia.*

Необходимостта от информация за особеностите на численото програмиране налага да се добавят към старите понятия характеризиращи ПИ регулиране допълнителните понятия от теорията на числата за настройване на програмиран в контролер ПИ регулатор. Авторът използва модерната алгебра за изясняване на критериите за настройване на пропорционалната част на числен ПИ регулатор. Така се изяснява строго математически понятието „инвариантно управление" и методите за прилагането му.

Introduction

The problem considered in the development is the choice of setting the PI controller according to the characteristics of the object it controls. For this purpose, a mathematical description of analog PI regulation in the field of real numbers was performed. This description is necessary to make a connection between the familiar analog regulators and the numerically functioning regulators programmed into controllers.

It is known from the help information (help) in the applications for programming controllers through examples of setting up PI controllers in a system of standard objects (see the list of literature below). However, the connection between the programmable controller and the "classic" PI controller is not apparent.

The topic of programming a PI regulator in a controller is extremely relevant today, because complex and expensive production installations are entering practice, the control of which by "classical" methods will, in principle, be expensive and complicated, because the "classical" control technique consists of numerous assembly units that are difficult to relate informatively to each other.

To compile the text, the author is used his own work "Theory of Invariance" [1], informational materials of the Siemens company about the controllers from the Simatic [3] and Logo [2] systems, as well as data from the Wikipedia internet encyclopedia.

In [4] the PI and Proportional Integral Derivative (PID) regulators are described and new methods for optimal tuning of analog PID controllers are derived, together with generalized discretization method, which is used to derive universal digital PID Controllers.

The main goal of the article is to understand the informational (mathematical) principle of numerical PI regulation when setting PI regulators in controllers. It is most important that the adjustment achieve stable operation of the "object-regulator" system during a transient process.

A. Invariant numerical control

Figure 1 shows the diagram of a system for automatic control of the output (production) analog value *x* of object O according to its programmed value x^0. The system is powered by the source *ES* and controlled by the proportional and integral (PI) regulator *P+I*. The system will be invariant if:

- the characteristic parameters of the object *O* and the regulator *P+I* are constant in time *t*;
- the power of the source *ES* is infinite.

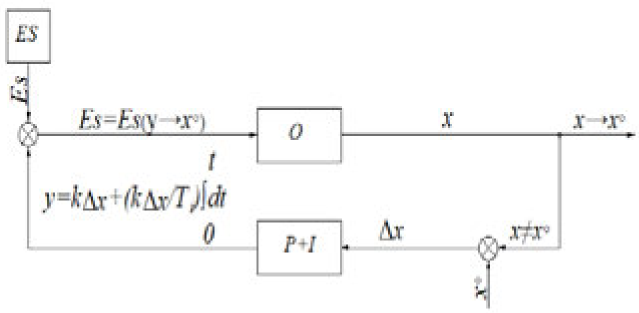

Fig. 1. Analog system "object - regulator"

Definition O1: The regulation of the system will be numerical (discrete, digital) if the analog output control function $x \rightarrow x^0$ is the result of a digital to analog conversion performed by the *P+I* regulator.

Definition O2: The object *O* represents an inertialess proportional unit of the system, which has a proportionality factor k_O allowing degrees of setting up, i.e.,

(1) $k_O = const, \{k_{O1,} k_{O2}, k_{O3}, \dots, k_{Oi}, \dots\}$

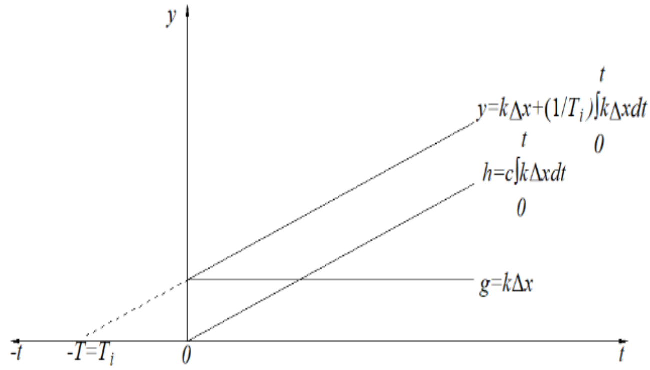

Fig. 2. Diagram of PI functions of the regulator.

B. Performance of a PI regulator.

The characteristic parameters of the regulator as indicated in figure 1 are its proportionality coefficient k and its integration time constant T_i. Their physical meaning can be explained with the following example.

Let it be assumed (see Figure 2.) that at the moment t=0 the coefficient of proportionality of the object k_O changes its value from k_{O1} to the smaller k_{O2} or, if

(2) $k_O = k_{O1}, t < 0,$ it is assumed that at

(3) $k_O = k_{O2}, t \geq 0, k_{O1} > k_{O2}$

This will cause the value of the output variable *x* to decrease from x^0 to x^0-Δx. As a consequence of this, according to Figure 1., the setting part of the regulator will automatically generate the difference $x^0 - (x^0 - \Delta x)$, which, as a result, will provide the computing part of the regulator with the positive control signal Δx.

In turn, the proportional part of the regulator will generate the control component

(4) $g = k\Delta x$

and the integral part – the control component

(5) $h = c \int_0^t k\Delta x dt$

Graphically, the above equation (5) represents, as depicted in Figure 2., lines with a angular coefficient *c*. This coefficient is equal to the angular coefficient of the line parallel to it

(6) $y = g + h$

And the angular coefficient represents the ratio

$$(7)\quad \frac{k\Delta x}{T_i} = c$$

The segment along the abscissa axis

$$(8)\quad (-T, 0) = T_i$$

in turn, represents a parameter set in the controller, which is called the integration time constant.

If in equation (5) the value of the angular coefficient c is replaced by relation (7), the equation of the integral line h, which represents the integral function of the regulator, will take the form:

$$(9)\quad h = \frac{k}{T_i}\int_0^t \Delta x dt = \frac{k\Delta x}{T_i}\int_0^t dt$$

And if in equation (6) the summation of g and h with the values from equations (4) and (9) is carried out, the equation of the line y, which represents the proportional and integral function of the regulator, will take the form

$$(10)\quad y = k\Delta x + \frac{k\Delta x}{T_i}t = k\Delta x(1 + \frac{1}{T_i}t)$$

This function will cause according to figure 1. the energy source ES to generate the energy flow to the regulated object O

$$(11)\quad Es = Es(y \rightarrow x^0)$$

so that the system reaches equilibrium

$$(12)\quad x = x^0, t \rightarrow \infty,$$

which the analog PI regulator will aim to achieve.

C. Numerical PI integration.

If in equation (10) time t is divided into equal intervals:

$$(13)\quad (t_0, t_1), (t_1, t_2), \dots, (t_i, t_{i+1}) \dots$$

the proportional and integral function of the regulator in equation (10) will divide into the enumerable set of functions:

$$(14)\quad y_0, y_1, y_2, \dots, y_i, y_{i+1} \dots$$

and each of them will have the form:

$$(15)\quad y_i = k\Delta x(1 + \frac{1}{T_i}(t_{i+1} - t_i))$$

Eventually the function y will convert to the sum:

$$(16)\quad y_0+y_1+y_2+\dots+y_i+y_{i+1},\dots$$

If, under the influence of the first regulation pulse y_0, the "object - regulator" system is driven to equilibrium, it means that equalities (10), (11) and (12) are true. From this it will inevitably follow that the proportional and integral function of the regulator y will decrease over time t its impact on object O. In the presence of this effect, the functional order (14) will be convergent, i.e.,

$$(17)\quad y_0>y_1>y_2>\dots>y_i>y_{i+1},\dots$$

and the sum (16) will be finite.

In order for the equality (16) to have a finite sum and the inequality (17) to be true, it is necessary to setting up the proportional part of the regulator k, the time constant of its integral part T_i, as well as the length of the integration intervals (13).

Theorem T1. In order to achieve the truth of the inequality (17), it is necessary that each regulation pulse y_i be smaller than the increment Δx_i of the output signal x of the system, i.e.,

$$(18)\quad y_i < \Delta x_i, t = t_i.$$

Proof: Let it be assumed that the controller is set so that the direction of inequality (18) for the first integration interval is reversed, i.e.,

$$(19)\quad y_0 > \Delta x_0, t = t_0.$$

Then at the moment $t=t_1$ the output quantity x of the system will increase, reaching the value $x^0+\Delta x_1$. In this case, the increase Δx_1 will be in antiphase with the decrease Δx_0, which will force the regulator to generate the pulse $-y_1$, which is antiphase to the pulse y_0.

This will lead, in turn, at the moment $t=t_2$ to a decrease of the output quantity x to the value $x^0-\Delta x_2$, and the decrease $-\Delta x_2$ will be antiphase to the increase Δx_1. So, at the starting moments of each regulation pulse of the sequence (row)

$$(20)\quad t_0, t_1, t_2, \dots, t_i, t_{i+1}, \dots$$

the regulator will generate pulses forming the sequence:

$$(21)\quad y_0, -y_1, y_2, -y_3, \dots, y_i, -y_{i+1}, \dots$$

which are equal in magnitude (module) and width, but in pairs opposite in phase to each other. Thus, the sequence of regulation pulses will cause the sequence of magnitude errors at the output of the system x

$$(22)\quad \Delta x_1, -\Delta x_2, \Delta x_3, \dots, \Delta x_i, -\Delta x_{i+1}, \dots$$

which are the same in module and width but by pairs opposite in phase to each other.

The system is stably oscillating and is unable to achieve the equilibriump predicted by equations (11) and (12).

If it is assumed that each control pulse of the controller is equal to the deviation Δx, it can be assumed that the first control pulse is

$$(23)\quad y_0 = \Delta x_0, t = t_0,$$

Condition (12) will indeed be achieved, but the state of the system will be critical because practically it is achieved by chance, independent of any human activity. Such a state can be accepted as stable equilibrium only in static conditions, while the system is accepted a priori - in dynamic ones.

If it is assumed that each regulation pulse of the regulator is less than the deviation Δx, it can be assumed that the first regulation pulse is less than the deviation Δx_0, i.e.,

$$(24)\quad y_0 < \Delta x_0, t = t_0,$$

Then at time t_0 the output quantity x^0 of the system will increase, reaching the value $x_1 = x^0 + \Delta x_1$. In this case, the increment Δx_1 will be less than the increment Δx_0, but in phase with it. And, if equilibrium is still not reached, Δx_1 will force the regulator to generate the pulse y_1, which will be smaller than the pulse y_0, but also in phase with it.

So, at the starting moments of each regulation pulse of the series (20), the regulator will generate pulses forming the inequality:

$$(25)\quad y_0 > y_1 > y_2 >, \dots, > y_i > y_{i+1}, \dots$$

Thus, the monotonically converging series of control pulses (24) will cause the series of magnitude errors at the output of the system x

$$(26)\quad \Delta x_0 > \Delta x_1 >, \dots, > \Delta x_i > \Delta x_{i+1}, \dots$$

which is also monotonically convergent and therefore the system will move towards the required equilibrium described in equations (11) and (12).

Theorem T1 is thus proved.

Thus, the analog quantity x at the output of the object O is numerically controlled with the required smoothness determined by setting up the regulator parameters.

D. Setting the integration intervals, the proportionality coefficient k_P and the time constant T_i of the regulator.

The condition (24) in which the first regulation pulse of the regulator will trigger the sequence of pulses (25) smoothly leading the system to a stable equilibrium is achieved by suitably adjusting the mentioned parameters of the regulator.

The integration (sampling) interval is selected according to the frequency capabilities of the regulator. So, for example, according to the possibilities of the controllers from the SIMATIC system of the Siemens Company, in analog regulation mode, it can be selected from 5 milliseconds with an unlimited upper limit.

The setting up of the frequency regulating integration interval determining its length is determined relative to the frequency of change of the coefficient k_O of proportionality of the object O described in equation (2). This frequency, in turn, is determined by statistical data from experimental tests of the object carried out by the company that produced or built it.

For objects where the output variable x changes slowly over time t, Siemens offers the controllers from the LOGO! System, where the integration interval is factory-set to 500 milliseconds. This factory setting is suitable for analog regulation in heating installations, pump and compressor stations, etc.

The integration time constant T_i of the regulator is determined relative to the transient time constant of the object O, which, in turn, is determined by statistical data from experimental tests of the object. It is different for different types of objects.

The setting up of the proportionality coefficient k of the regulator is carried out according to the requirement that each regulating pulse y_i be smaller than the deviation Δx_i causing it at the time t_i. For this purpose, it is necessary, first of all, to determine at what magnitude of the regulating pulse y_i, setting the proportionality coefficient k of the regulator will put the system in the critical state determined by equality (23). The value of the coefficient k that led to this condition is the maximum allowable or limit value. It serves as a measure when choosing the settings guaranteeing the security of the system.

The set of regulation pulses (25) represents the numerical algebraic one structure multiplication group. A measure of evaluating each member of the group and comparing it to each of the other members is the number one. Therefore (see Figure 1.), if the unit increment of the system output Δx causes the regulator setting device to supply to its input the signal Δx_0, which causes the regulator to generate the unit pulse y_0 , i.e.,

$$(27)\ \Delta x_0 = 1 \rightarrow y_0 = 1, t = t_0,$$

then equality (15) will take the form:

$$(28)\ 1 = k(1 + \frac{T_S}{T_i}), \text{като } T_S = t_0 - t_1,$$

from which it will follow that the limit value k_m, to which it is permissible to reach the magnitude of the proportionality coefficient k of the regulator is

$$(29)\ k_m = \frac{1}{1+\frac{T_S}{T_i}} = \frac{T_i}{T_i+T_S}$$

and this means that each value of this coefficient must be

$$(30)\ k < k_m$$

to ensure a stable smooth proportional and integral regulation of the production quantity x of the system.

E. Limits of invariance

This part of the paper defines the boundary conditions under which the “object-regulator” system will be invariant.

Figure 3. shows the statue of the Jewish King David, which is a copy of the original work by Michelangelo Buonarroti. The statue is 5,17 meters tall and weighs 5,56 tons. After its creation in 1504 until 1873 it stood in the Palazzo della Signoria in Florence, where its copy stands today. From 1873 until today and in the future, the statue has stood and will stand indoors, in a suitable climate, in the Florentine museum Galleria dell' Accademia.

Both statues are sculpted from marble quarried in Carrara, where in the era of the Roman Empire countless disenfranchised slaves left their bones to reveal today's randeman.

In 2003-2004, the marble in the museum was carefully cleaned and regularly dusted to ensure a real view of Michelangelo's creation.

Fig. 3. Copy of Michelangelo's David.

So, a statue with dimensions and weight far from infinite and an outdoor life of some 369 years, which is far from eternal - although sculpted from the most precious marble to mankind - turns out to have extremely limited invariants. And they, the invariants want human care in order to have them.

In homage to the genius of Michelangelo, although his dream of endless and eternal admiration for the creation of the hand remained unfulfilled his work of art, as well as with expressed respect for the memory of the slaves, who died to achieve even today's quarrying of Carrara marble, let us accept as true the following

Axiom A1. Nothing materially created by human hands is infinite in space and eternal in time.

This axiom outlines the constraints on the invariance state of any system or facility. Based on it follows

Theorem T3. Invariant can be any system or facility that has been tested under extreme conditions (load, temperature, pressure, energy consumption, vibration, etc.) before starting their operation according to established legal rules. Moreover, these extreme conditions can never and nowhere be reached under the prescribed operating rules.

Proof: If it is assumed that, it is possible for the parameters of the systems and facilities to remain constant outside the operating rules regulated for them, it will turn out that axiom A1 is not true.

This proves the theorem.

Conclusion

PI regulation is the most applied in practice. A very small proportion of installed PID controllers use an included differential part. And the mass entry of controllers into practice requires computer specialists to know both production technologies and their management in PI mode.

The contribution of the article is the criterion for setting the proportional part of the PI controller in the presence of selected integration time constant and integration intervals.

The author recommends that the future followers dealing with the regulation of production processes should repeatedly experiment with the above criterion in order for it to be realistically evaluated.

Many authors have theoretically worked on the problems of PI regulation, given its mass application in practice, with the aim of optimizing the work of the "object - regulator" system. The optimum was sought by the means of mathematical description of various types of objects, in which hard-to-apply materially abstract solutions were achieved.

This proves that optimal solutions of PI regulation exist only in the conditions of physically admissible states of invariance of the "object-regulator" system.

REFERENCES

[1] Stankov M., Theory of Invariant Automatic Control, IP address: http://framework.com/stankov/

[2] Logo! Soft Comfort V8.2. Help (Siemens product for programming the controllers from the system Logo). https://support.industry.siemens.com/cs/document/109767413/logo!-soft-comfort-v8-2-sp1-(download-notes instructions)?dti=0&lc=en-BG

[3] TIA Portal V18. Help (Siemens product for programming the controllers from the system Simatic). https://www.pccweb.com/2023/04/05/siemens-tia-portal-v18/

[4] Tomov L. Analog and Digital PID Controllers synthesis for optimal control of complex (linear) time delay systems. Thesis for: PhD, 2011. https://www.researchgate.net/publication/301747214_Analog_and_Digital_PID_Controllers_synthesis_for_optimal_control_of_complex_linear_time_delay_systems_In_Bulgarian (In Bulgarian)

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