Some features for calibration of analyzers of electric power by power factor

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In relation to the calibration of analyzers of electric power, the mathematical model of the power factor is represented by its dependence on the phase angle of voltage and current of the first harmonic as well as on the general harmonic distortion of voltage and current. A metrological analysis of the power factor deviation was performed, depending on the voltage and current parameters generated by the reference calibrator.

Keywords – analyzer of electric power, calibration, power factor.

Някои особености при калибриране на анализатори на електрическа енергия по фактор на мощността (Пламен М. Цветков, Красимир С. Гълъбов, Иван Н. Коджабашев). *За целите на калибрирането на анализатори за оценка на качеството на електрическата енергия по фактор на мощността се анализира математическия модел на параметъра фактора на мощността и зависимостта му от фазовия ъгъл на напрежението и тока на първия хармоник, както и от коефициентите на нелинейни изкривявания на напрежението и тока. Направен е метрологичен анализ на отклонението на фактора на мощността като функция на параметрите на напрежението и тока, генерирани от еталонния калибратор.*

Ключови думи: анализатор на електрическа енергия, калибриране, фактор на мощността.

Introduction

One of the parameters for calibration and traceability of the metrological characteristics of analyzers of electric power is the power factor, which is a function of all the harmonic components of the voltage and current and the phase angle of the voltage and current of the first harmonic. It is interesting to define a mathematical model and evaluate the influence of the voltage and current parameters on the deviation of the power factor set by a calibrator.

Calibration by power factor and mathematical model

In calibrating of the electrical energy analyzer, referred further as the analyzer, the method of comparison of the measuring instrument with the traceable standard (reference calibrator, shortly - a calibrator), corresponding to the requirements of the traceability chain according to the block diagram of Fig.1 is applied.

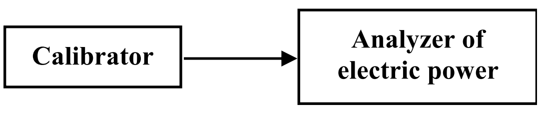

Fig.1. Scheme of calibration.

The calibration process determines the actual values of the corresponding power factor. In the calibration the relationships between the reading of the analyzer and the realized by the calibrator power factors are established.

The mathematical model for the estimate of the actual effective value of the power factor (PF), according to [1] is the following:

(1) $$PF_{act} = PF_{cal} - \delta PF_{et} + \delta PF_{res.cal},$$

where:

- PF_{cal} - is the measured effective value of the power factor obtained by the calibrated analyzer, using in repeated measurements the estimate $PF_{cal} = \frac{1}{k}\sum_{i=1}^{k} PF_{cal,i}$, determined as an average value of the individual observations (measurements) $U_{n,cal,i}$

- k - is the number of measurements, usually selected $k \geq 10$
- i - is the measurement index
- $\delta PF_{res.cal}$ - is the correction of the measured value of the PF, due to the resolution of the calibrated analyzer
- δPF_{et} - is the correction of the set by the calibrator power factor, which is generally due to different reasons such as:

– $\delta PF_{s.et}$ - a deviation of the set (generated) value of the calibrator due to combined effects of offsets, non-linearity, and other instrumental and methodic imperfections of the calibrator. This deviation can be determined from the calibrator technical documentation (while the calibrator is not calibrated) or from its calibration certificate as a correction of the calibration point ;

– $\delta PF_{dr.et}$ - a drift of the generated by the calibrator value, compared to the calibrator's last calibration (a drift since its last calibration);

– $\delta PF_{t.et}$ - a deviation of the calibrator value as a result from a change in the environment temperature;

– $\delta PF_{l.et}$ - a deviation of the calibrator value due to changes in the supply voltage;

– $\delta PF_{z.et}$ - a deviation of the calibrator as a result from the energy exchange of the calibrator due to the input impedance of the calibrated analyzer.

In calibration of a particular analyzer only these components should be considered which have the most significant contribution to the correction δPF_{et}.

The deviation of the calibrator set point value of the power factor of the calibrator $\delta PF_{s.et}$ is determined by the relationship

(2) $$\delta PF_{s.et} = \delta_{PF,et} PF,$$

where:

- PF is the effective value of the PF set by the calibrator,
- $\delta_{PF,et}$ is the relative error of the calibrator.

In calibration of the analyzer using a power factor by the help of the calibrator, the nominal effective voltage value of the main (first) harmonic $U_{1nom} = 230V$ and an effective value for the corresponding (relevant) voltage, are set. This approach can be applied only if the calibrator has the ability to generate periodic signals of voltage and current, which is the sum of two or more harmonic signals, one of which is the main (first) harmonic with a frequency $f = 50Hz$.

Mathematical model of power factor

It is known that current, voltage and active power can be described by the following dependencies [2]:

- for voltage

$$u(t) = \sum_{n=1}^{\infty} U_{nm} \sin(n2\pi ft + \rho_n);$$

- for current

$$i(t) = \sum_{n=1}^{\infty} I_{nm} \sin(n2\pi ft + \theta_n);$$

- for active power

$$P_{avr} = \frac{1}{T}\int_0^T u(t)i(t)dt =$$

$$= \frac{1}{T}\int_0^T \sum_{n=1}^{\infty} U_{nm} I_{nm} \sin(n2\pi ft + \rho_n)\sin(n2\pi ft + \theta_n)dt =$$

$$= \sum_{n=1}^{\infty} \frac{1}{T}\int_0^T \frac{U_{nm} I_{nm}}{2}\cos(\rho_n - \theta_n)dt =$$

$$= \sum_{n=1}^{\infty} \frac{1}{T}\int_0^T \frac{U_{nm}}{\sqrt{2}}\frac{I_{nm}}{\sqrt{2}}\cos\varphi_n dt =$$

$$= \sum_{n=1}^{\infty} U_{nrms} I_{nrms}\cos\varphi_n = \sum_{n=1}^{\infty} S_n \cos\varphi_n = \sum_{n=1}^{\infty} P_{navr}$$

where U_{nm} and I_{nm} are the maximum values of harmonic voltages and currents

- U_{nrms} и I_{nrms} are the effective values of harmonic voltages and currents,
- f is the frequency of the main harmonic
- n is the number of the corresponding harmonic
- ρ_n и θ_n are the initial phase angles of harmonic voltages and currents,
- φ_n is the phase difference of voltage and current respectively,
- P_{avr} and P_{navr} are the active power of the load and the active power of the respective harmonics,
- S_n is apparent power of the respective harmonics.

The indexes avr and rms refer to the average value and the effective value respectively. Then the active power can be expressed as

(3)
$$P=\sum_{n=1}^{\infty}U_nI_n\cos\varphi_n=\sum_{n=1}^{\infty}P_n=\sum_{n=1}^{\infty}S_n\cos\varphi_n= =S_1\cos\varphi_1+\sum_{n=2}^{\infty}S_n\cos\varphi_n=P_1+\sum_{n=2}^{\infty}P_n,$$

The relationships of the total harmonic distortion (THD) with voltage and current are also known

(4)
$$THD_U=\frac{\sqrt{\sum_{n=2}^{\infty}U_n^2}}{U_1}\ \text{и}\ THD_I=\frac{\sqrt{\sum_{n=2}^{\infty}I_n^2}}{I_1}$$

as well as the relationships between effective voltage and current values through the load

(5)
$$U=\sqrt{\sum_{n=1}^{\infty}U_n^2}=U_1\sqrt{1+THD_U^2}$$

and

$$I=\sqrt{\sum_{n=1}^{\infty}I_n^2}=I_1\sqrt{1+THD_I^2}$$

Thus, for the power factor [3], the following expression can be written

(6)
$$PF=\frac{P}{UI}=\frac{P_1+\sum_{n=2}^{\infty}P_n}{U_1I_1\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}= =\frac{P_1}{U_1I_1\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}+ +\frac{\sum_{n=2}^{\infty}P_n}{U_1I_1\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}$$

Practice proves that in most cases the value of the active power P is due only to the active power of the main (first) harmonic, i.e. $P=P_1$ and $\sum_{n=2}^{\infty}P_n\approx 0$. Then it can be written:

(7)
$$PF=\frac{P}{UI}\approx\frac{P_1}{U_1I_1\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}= =\cos\varphi_1\frac{1}{\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}=PF_{disp}\cdot PF_{dist},$$

where:

- $PF_{disp}=\cos\varphi_1$ is the power factor of the phase difference (phase shift / dephasing) of the current and voltage of the main (first) harmonic and
- $PF_{dist}=\frac{1}{\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}$ is the power factor of the non-linear distortion due to the presence of harmonics.

From the obtained expression (7) it can be concluded that the value of the power factor generated by the reference calibrator does not need to be set, but can be calculated from the set values $\cos\varphi_1$, THD_U и THD_I , i.e. the power factor is set indirectly.

Note: In most practical cases $THD_U<10\%$, which means that $\frac{1}{\sqrt{1+THD_U^2}}\approx 1$. This leads to the simplified expression for the power factor $PF=\frac{P}{UI}=\cos\varphi_1\frac{1}{\sqrt{1+THD_I^2}}$. For calibration purposes, this expression is only applicable if the voltage consists of a main harmonic.

Analysis of error in calibration using power factor

In this case, it is interesting to study how the error is influenced by the error in setting the values of $\cos\varphi_1$, THD_U и THD_I. To determine their influence it is necessary to find the full differential of function (7).

After differentiating of (7) the variable μ obtained

(8)
$$dPF=\frac{\partial PF}{\partial\varphi_1}\Delta\varphi_1+\frac{\partial PF}{\partial K_U}\Delta THD_U+\frac{\partial PF}{\partial K_I}\Delta THD_I= =\left(-\frac{1}{\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}}\sin\varphi_1\right)\Delta\varphi_1+ +\left[-\frac{\cos\varphi_1}{\sqrt{1+THD_I^2}}\frac{THD_U}{\sqrt{1+THD_U^2}\left(1+THD_U^2\right)}\right]\Delta THD_U+ +\left[-\frac{\cos\varphi_1}{\sqrt{1+THD_U^2}}\frac{THD_I}{\sqrt{1+THD_I^2}\left(1+THD_I^2\right)}\right]\Delta THD_I,$$

From (8) the relative error of the reference calibrator is obtained

(9)
$$\delta_{PF,et}=\frac{dPF}{PF}= =-\varphi_1\tan\varphi_1\delta_{\varphi_1}-\frac{THD_U^2}{\left(1+THD_U^2\right)}\delta_{THDU}-\frac{THD_I^2}{\left(1+THD_I^2\right)}\delta_{THDI}= =a_{\varphi_1}\delta_{\varphi_1}+a_{THDU}\delta_{THDU}+a_{THDI}\delta_{THDI}$$

where:

- δ_{φ_1}, δ_{THDU} and δ_{THDI} indicate the relative errors to setting the phase angle respectively of voltage and current of the first harmonic, of the total harmonic distortion of the voltage and of the total harmonic distortion of the current.
- a_{φ_1}, a_{THDU} и a_{THDI} - the coefficients of influence of the respective relative errors

Using the expression (9) and the relationship (2), for the deviation of the power factor set by the reference calibrator it is obtained

(10)

$$\delta PF_{s.et} = \delta_{PF,et} PF =$$

$$= \left(a_{\varphi 1}\delta_{\varphi 1} + a_{THDU}\delta_{THDU} + a_{THDI}\delta_{THDI} \right) PF =$$

$$= \left(\cos\varphi_1 \frac{1}{\sqrt{1+THD_U^2}\sqrt{1+THD_I^2}} \right) *$$

$$* \left(-\varphi_1 \tan\varphi_1 \delta_{\varphi_1} - \frac{THD_U^2}{\left(1+THD_U^2\right)}\delta_{THDU} - \frac{THD_I^2}{\left(1+THD_I^2\right)}\delta_{THDI} \right)$$

Conclusions

1. The parameters that determine the power factor setting error are $\cos\varphi_1$ of the main harmonic and the coefficients of total harmonic distortion of voltage THD_U and current THD_I.

2. The calibration by power factor can be combined with the calibration process by total harmonic distortion and $\cos\varphi$.

3. If by calibration are used two periodic squarewave bipolar signal, respectively for voltage and current, the sources of error are limited to the duty cycle of voltage μ_U and duty cycle of current μ_I and the error of setting the phase angle φ of the two pulse series.

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Received on: 20.08.2018