Design and investigations of approximately linear phase Hilbert transformers

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In this work a new design procedure of approximately linear phase (ALP) digital Hilbert transformers (HT) is proposed. First, the possible positions of the transfer function (TF) poles of such HTs realizations are studied and then different allpass sections, realizing such TF poles are investigated. A very efficient approach to extend the frequency range over which the phase difference of the approximately linear phase allpass-based digital HT is kept near constant while reducing the deviation of the phase from 90º in a limited word-length environment is proposed. It is based on a worst-case phase-sensitivity minimization of each individual special second and fourth order allpass sections in the cascade realization of the imaginary branch of the ALP HT, including replacements of some special second order allpass sections by two real first order low sensitivity allpass sections. The effectiveness of the proposed design is experimentally verified and the HTs so obtained are very suitable for different telecommunication applications, especially in the situations where the energy consumption has to be reduced.

Проектиране и изследване на Хилбертови трансформатори с приблизително линейна фаза (Камелия Николова). *В тази статия е предложена нова процедура за проектиране на цифрови Хилбертови трансформатори (ХТ) с приблизително линейна фаза. Първо са анализирани всички възможни местоположения на полюсите на предавателната функция (ПФ), характерни за този тип ХТ и след това са изследвани различни фазови звена (ФЗ), които реализират тези полюси. Предложен е много ефективен подход, който се основава на минимизиране на фазовата чувствителност на всяко отделно специално фазово звено от втори и четвърти ред, участващо в каскадната реализация на имагинерния клон на цифров ХТ с приблизително линейна фаза, като включва и замяна на някои от специалните фазови звена от втори ред с две реални ниско-чувствителни фазови звена от първи ред. В резултат на този подход се разширява честотният обхват, при който цифровият ХТ с приблизително линейна фаза запазва фазовата разлика приблизително постоянна и се намалява отклонението от 90º на фазовата разлика при работа с ограничена дължина на кодовата дума. Ефективността на предложената процедура за проектиране е доказана експериментално и така получените Хилбертови трансформатори са много подходящи за различни телекомуникационни приложения, особено в случаите, когато потреблението на енергия трябва да се намали.*

I. Introduction

Hilbert transformers (HT) find numerous applications in different fields of telecommunications and signal processing. They are important building blocks in radar and sonar systems, in single sideband modulation, in speech/audio/image/video processing. HTs are used for *I* and *Q* signal generation [1], for correction of the distortions in the loudspeakers [2], for suppression of oscillations in acoustic feedback systems [3] and in many other practical systems. A number of methods for designing digital HTs have been developed through the years and most of them have been well systematized in [4] [5] [6]. The digital HTs can be realized either with finite impulse response (FIR) or infinite impulse response (IIR) filters. The FIR based HTs are well described and investigated in [7]. Their disadvantages like very high transfer function (TF) order, total delay and power consumption are not acceptable for portable and mobile telecoms applications and equipments. That is the reason the IIR realizations to be preffered, although the possible instability can occur in a case of limited worldlength environement and severe TF coefficients quantization. Most often the IIR Hilbert transformers are based on the usage of allpass structures. The theory of the allpass-based HTs is quite mature and several design methods using real or complex allpass structures have been summarized in [4] [5] [6]. The HT design procedure depends on the initial halfband (HB) design method selected (even- or odd-order, approximately linear-phase or minimum phase).

The approximately linear phase (ALP) allpass-based HTs are chosen to be considered in this paper. This type of HT consists of allpass structures only in the imaginary branch while in the real one only the time shifts are performed. Although, the complexity of the allpass structures is higher than in the nonlinear-phase case, the ALP allpass-based HT is used in numerous practical applications, especially in the cases when the delay of the liner-phase FIR counterpart cannot be tolerated [8]. One of the advantages of this structure is that the TF poles are not so closed to the unit circle which leads to better roundoff noise performance and lower coefficient sensitivity, especially in narrowband case [9]. The variety of design methods of IIR-halfband filters presented for systems with approximately linear phase for maximally flat and Chebyshev approximations are well summarized in [5], and some new design procedures are proposed in [10] [11], but no specific methods for accuracy improvement have been reported. The problem with the accuracy of the realization of the HTs is very important in many of the telecommunication applications, like in mobile communications, WiMAX and software-defined radio. In many of these applications and especially in portable and mobile communication equipment a fixed-point arithmetic with a shorter wordlength is used. This results in accuracy reduction and special measures have to be taken to prevent that.

The main aim of this work is to try to improve the accuracy of the ALP allpass-based HTs throughout the phase sensitivity minimization of the individual allpass sections used in the cascade realization of the imaginary branch. As a result, a further reduction of the computational load, achievement of shorter word length and lower power consumption for a given accuracy are expected. The design procedures, including such sensitivity minimization should be straightforward, without iterative and complicated optimization steps and must keep low the complexity and the TF order.

This paper is organized into 7 sections. Section 2 summarizes the allpass-based ALP HTs design procedure. In Sections 3 the realization of the initial and special allpass sections are considered. The worst-case phase-sensitivities are studied in section 4. The proposed low-sensitivity design procedure is outlined in Section 5. Based on the knowledge gathered from our previous investigation [12] [13] an optional design step is included in the proposed design procedure - to further minimize the sensitivity by replacing some of the special second order allpass sections by cascades of properly selected first order allpass sections. In Section 6 it is demonstrated experimentally how efficiently the accuracy of the HTs in a limited wordlength environment is improved. Concluding remarks are given in Section 7.

II. Approximately linear phase design

An ideal digital Hilbert transformer is described in the frequency domain as [14]

$$H_{HT}(e^{j\omega}) = \begin{cases} -j, & 0 \le \omega < \pi \\ j, & -\pi \le \omega < 0 \end{cases}. \qquad (1)$$

HT is a type of digital filter whose purpose is to introduce a 90° phase shift of the input signal. In the ideal HT all the positive frequency components are shifted by -90°, while all the negative components are shifted by 90° [7]. Unfortunately, this ideal system is non-causal and cannot be realized. Thus, all practical realizations are only approximating an ideal HT, having non-constant 90^o phase-response over a frequency range narrower than ($0\le\omega<\pi$) and non-constant unity magnitude-response (in the case of all FIR and some IIR realizations). Every known allpass-based HT design method starts from a HB filter. The following relations between the passband Fp and the stopband F_{SB} edges and between the passband δp and stopband δ_{SB} ripples plus the admissible phase deviation $\Delta\varphi_{max}$ are valid for the prototype HB filter [4]

$$\delta_p = 1 - \sqrt{1-\delta_{SB}^2}; \qquad (2a)$$

$$F_p = 0.5 - F_{SB}; \qquad (2b)$$

$$\delta_{SB} = \sin(\Delta\varphi_{\max}/2). \qquad (2c)$$

The design of ALP allpass-based HT starts from a special IIR halfband filters with the following transfer function [4] [6]:

$$G(z) = 0.5[z^{-N} + A(z^2)], \qquad (3)$$

where N is restricted by the condition, that the total degree of the system has to be odd [15]. Thus, the allpass TF $A(z^2)$ must be of even order chosen to be $N \pm 1$. This results in increasing the complexity of the

$A(z^2)$ and thus the allpass TF could be represented as a product of second-order $A_{2k}(z^2)$ and fourth order $A_{4k}(z^2)$ terms [7] as it is given in [5][6]

$$A(z^2) = \prod_{k=1}^{K_{0,1}} A_{2k}(z^2) \prod_{k=1}^{K_{0,2}} A_{4k}(z^2) = \prod_{k=1}^{K_{0,1}} \frac{a_k + z^{-2}}{1 + a_k z^{-2}} \prod_{k=1}^{K_{0,2}} \frac{c_k + b_k z^{-2} + z^{-4}}{1 + b_k z^{-2} + c_k z^{-4}} . \quad (4)$$

The specific issue here is the initial HB TF $G(z)$ poles positions. The poles are located either on the imaginary axis or symmetrically around the imaginary axis, but far from the unit circle as it is shown in Fig. 1 for the case of 14th order HB filter ($K_{0,1} = 1$ and $K_{0,2} = 3$). It is demonstrated also how these poles are assigned to the individual 4th order TFs, respectively $A_{41}(z^2)$, $A_{42}(z^2)$ and $A_{43}(z^2)$.

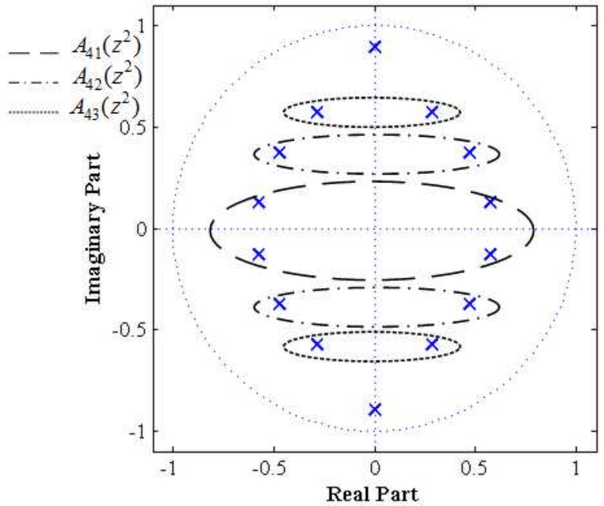


Fig. 1. HB TF pole positions for ALP design

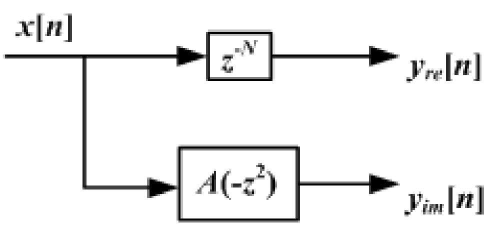


Fig. 2. ALP HT realization

The corresponding ALP HT has the following TF:

$$H(z) = 2jG(-jz) = \left[(-1)^{(N+1)/2} z^{-N} + jA(-z^2)\right]. \quad (5)$$

The realization (for real input signal $x(n)$) is given in Fig. 2.

III. Allpass sections realizations

As it can be seen from Fig. 1 and as follows from (4) special second-order and also special fourth-order allpass sections will be needed. We call them "special" because of the missing odd degrees of the polynomials in the numerator and denominator (4). All poles in case of the special second order allpass TF lie in couples on the real axes. These sections are having zero TF coefficients before z^{-1} and can be obtained from every known real first order allpass section by adding a unit delay in the delay circuit. The poles of the special fourth order section are appearing in quadruplets with two mirror-image (with respect the imaginary axes) couples of complex-conjugated poles. Similarly, these sections can be obtained from every known real second order allpass section by replacing every delay element by two delays.

The worst-case phase-sensitivity minimization of the individual allpass section used in the realization of every couple of complex-conjugated poles is considered. We have, therefore, to identify all possible TF poles positions and to analyze all known allpass sections in order to have a proper selection for every pole position.

A. First order allpass sections

Our investigation in [12] shows that there is no allpass section providing low sensitivity for the entire frequency range. It was found in [13] [16] that several low-sensitivity sections for every single real pole position could be found: the *ST*1 section (6), providing low-sensitivity for poles near z=1, *MH*1 (7), *SC* (8), having low sensitivity for poles near z = 0 and *SV* (9) section for poles near z = -1. Their transfer functions are:

$$H_{ST1}(z) = \frac{-(1-a) + z^{-1}}{1 - (1-a)z^{-1}}; \quad (6)$$

$$H_{MH1}(z) = \frac{-b + z^{-1}}{1 - bz^{-1}}; \quad (7)$$

$$H_{SC}(z) = \frac{-b - z^{-1}}{1 + bz^{-1}}; \quad (8)$$

$$H_{SV}(z) = \frac{1 - c + z^{-1}}{1 + (1-c)z^{-1}}. \quad (9)$$

B. Second order allpass sections

We have studied most of the known second order allpass sections in our previous research [13] [17]. A part of them like those proposed in [18] (*ST*2*A* and

ST2B) are developed in order to have low sensitivity for TF poles near $z = 1$. What is specific here is that the TF poles (4) are situated closer to the center of the unit circle what is not the usual case. The investigations of allpass based fractional delay digital filters in [17] have shown the need for development of allpass sections with low-sensitivity for pole position around the area z=0 and such second order allpass section (namely *IS*) was proposed there. It was also found in [17] that there is a deficiency of sections for low sensitivity realizations of poles in other zones of the unit-circle. We selected to use here *ST2A* and *IS* together with the most popular sections, having canonical structures and known with low sensitivities, namely the Mitra and Hirano sections (*MH2B*) and the Kwan sections (*KW2B*). These sections are realizing the following TFs:

(10) $$H_{MH2B}(z) = \frac{b_2 - b_1 z^{-1} + z^{-2}}{1 - b_1 z^{-1} + b_2 z^{-2}};$$

(11) $$H_{KW2B}(z) = \frac{d_1 + d_2 - 1 - (d_1 - d_2) z^{-1} + z^{-2}}{1 - (d_1 - d_2) z^{-1} + (d_1 + d_2 - 1) z^{-2}};$$

(12) $$H_{ST2A}(z) = \frac{1 - 2b - 2(1-b)(1-2a) z^{-1} + z^{-2}}{1 - 2(1-b)(1-2a) z^{-1} + (1-2b) z^{-2}};$$

(13) $$H_{IS}(z) = \frac{b + (-a - 2b + ab) z^{-1} + z^{-2}}{1 + (-a - 2b + ab) z^{-1} + b z^{-2}}.$$

C. Special second order allpass sections

The allpass TFs in (4) are having all their poles as it was shown in Fig. 1, while the poles of ALP HTs (5) are situated in mirror-imaged couples around and on the real axes. In the literature [7] the special second order allpass sections usually are realized starting from the well known lattice allpass section which coincides with the popular Mitra-Hirano *MH1* section (7). But our investigation shows that the possible real pole positions are scattered over the real axes and more especially in the range $\pm[0.5 \div 1)$ and thus it is expected that the special second order section derived from *MH1* section will have different phase sensitivities when realizing different TF poles. In order to improve the accuracy of the allpass-based HTs we propose to minimize the overall sensitivity in the lower branch in Fig. 1 by realizing every couple of real poles with second-order sections, providing the lowest sensitivity for the given TF pole positions.

We developed several special second-order sections starting from the real first order sections described in Section 3A by changing the signs of the coefficients of the allpass TFs in Eqs. (6)–(9) and by replacing z^{-1} by z^{-2} in the allpass structures [5][6]. We denote these second order allpass sections as *MH1-2*, *ST1-2*, *SV-2* and *SC-2*.

D. Special fourth order allpass sections

As it was mentioned above the special fourth order allpass TF in (4) can be obtained from every known second-order real allpass section by replacing z^{-1} by z^{-2} and z^{-2} by z^{-4} in the allpass TFs (10)-(13) i.e. by adding an additional delay element to the delays in their realizations. Usually, in the literature, these sections are realized starting with MH2 (10) sections. The possible poles positions of such special fourth order allpass TF have been studied for different ALP HT TF order and it was found that they are scattered over the entire unit-circle as it was the case for the allpass based fractional delay digital filters considered in [17], which means that most of the realizations with Mitra and Hirano sections will have quite high sensitivities. We can reduce these sensitivities by realizing every quadruplet of poles with a structure having the lowest sensitivity for the pole positions of every specific quadruplet.

We have developed several special fourth-order sections starting from the TFs (10)-(13) and we denoted these special fourth order allpass sections as *MH2B-4*, *KW2B-4*, *ST2A-4* and *IS-4*.

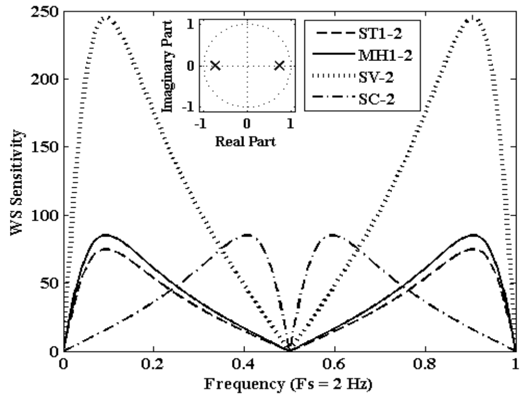
IV. Sensitivity investigations

A. Special second order allpass sections

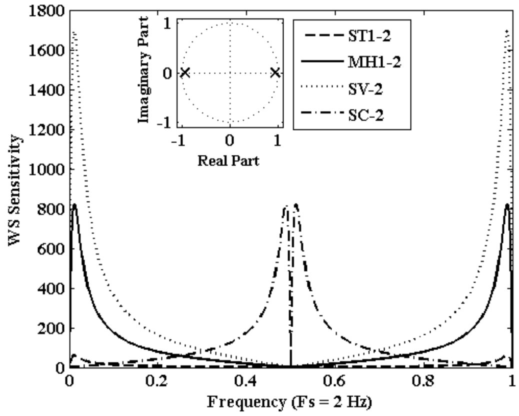
Two typical for approximately linear phase HTs real pole positions were selected:

- a couple of poles near $z = \pm 1$;
- a couple of poles around $z = \pm 0.5$.

The WS phase-sensitivities of the four special second-order sections from Sect. 2.C are given in Fig. 3 for TF pole positions, corresponding to coefficients $b_{MH1\text{-}2} = 0.53225$ and $b_{MH1\text{-}2} = 0.932734$. We calculate the WS phase sensitivity by using the package PANDA [19]. It can be seen that there is a significant difference between the maximal values of the sensitivities, and most obviously for the poles near $z = \pm 1$ - the difference is more than 10 times. As a result, we can conclude that the worst-case phase sensitivities of special second order allpass sections so derived conduct the behavior of those of the corresponding real second order allpass sections. It is obvious that the *ST1-2* is the best choice for TF poles (Fig. 3b) near $z = \pm 1$. But this is not the case for the other TF pole positions (Fig. 3a) when more than one section with similar sensitivities are possible, so additional investigations must be done in order to design the HT realization with the lowest sensitivity.



a) $b_{MH1\text{-}2} = 0.53235$, resp. a, $c = \pm 0.46765$

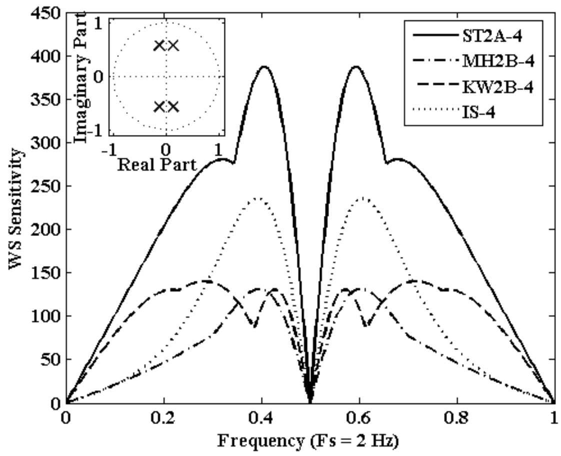


b) $b_{MH1\text{-}2} = 0.932734$, resp. a, $c = \pm 0.067266$

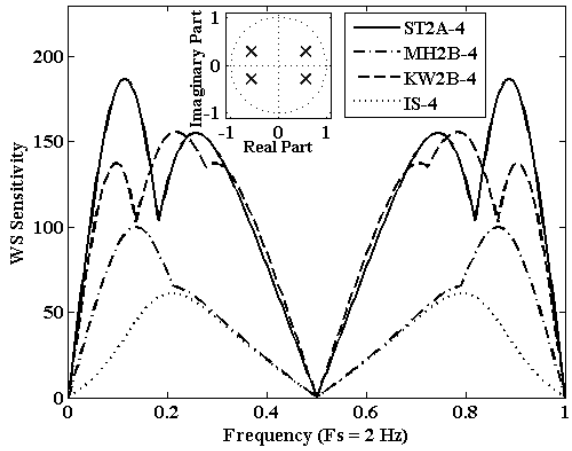
Fig. 3. WS phase-sensitivities of the special second-order allpass sections for two different TF poles positions.

B. Special fourth order allpass sections

In Fig. 4 the WS phase-sensitivities of four special fourth-order sections for two different TF poles positions are given. The corresponding TF poles positions are also given in Fig. 4.



a) $b1_{MH2B\text{-}4} = -0.627053$, $b2_{MH2B\text{-}4} = 0.120771$



b) $b1_{MH2B\text{-}4} = 0.489785$, $b2_{MH2B\text{-}4} = 0.166375$

Fig. 4. WS phase-sensitivities of special 4th order allpass sections.

As it can be seen, there is a great difference between the sensitivities of the individual realizations for the TF poles considered. The *KW2B*-4 and *MH2B*-4 phase sensitivities are from two to four times smaller compared to the others for TF poles positions given in Fig. 4a. The worst-case phase-sensitivities behavior is pretty different for the TF poles position given in Fig. 4b, where the *IS*-4 section has the best choice. Therefore, the proper selection of the allpass sections for such TF poles positions producing the ALP HT having the lowest sensitivity is not so clear and not so easy. It will be rather necessary to conduct some additional investigations for every quadruplet of poles in order to design the HT realization with the lowest overall sensitivity.

C. Overall HT sensitivity investigations

The overall WS sensitivity of the ALP HT depends only on the sensitivity of the allpass sections used in the cascade realization in the imaginary branch to the changes of all the multiplier coefficients m_k

$$WS_{m}^{\varphi_i(\omega)} = \sum_{k=1}^{K} \left| S_{m_k}^{\varphi_i(\omega)} \right|, \qquad (14)$$

where K is the number of the sections whose phase response is denoted by $\varphi_i(\omega)$. The WS sensitivity in every branch is calculated using PANDA [19].

In order to estimate how the proper choice of the special sections will affect the behavior of the ALP HT realization in a limited wordlength environment, an initial ALP halfband filter (4) was designed with Fp=0.3, N=13 and N_A=N+1=14, and with a Chebyshev approximation of the desired linear phase. The allpass TF $A(z^2)$ was obtained by following the programs described in [5] [6] and its poles positions are exactly these given in Fig. 1. The allocation of the poles to the corresponding sections' TFs is also given in the Fig.1. The design procedure results in 66 dB for the minimal stopband attenuation of the initial filter.

The TF poles of $A(-z^2)$ (5) are: $p_{1,2}$=±0.89317, $p_{3\text{-}6}$=±0.13037±j0.57491, $p_{7\text{-}10}$=±0.37132±j0.47199 and $p_{11\text{-}14}$=±0.57130±j0.28548 and they have to be realized as a cascade of one special second-order and three special fourth-order sections. Four realizations with different possible sets of special sections were designed. The combinations of the sections in these four realizations are given in Table 1 and the WS sensitivities of the imaginary branch (Fig. 2) are shown in Fig. 5.

Table 1

Different structures selections to realize every TF pole for ALP HT design

TF poles / Variants	$p_{1,2}$	$p_{3\text{-}6}$	$p_{7\text{-}10}$	$p_{10\text{-}14}$
1	*MH*1-2	*MH2B*-4	*MH2B*-4	*MH2B*-4
2	*MH*1-2	*KW2B*-4	*KW2B*-4	*KW2B*-4
3	*MH*1-2	*IS*-4	*MH2B*-4	*ST2A*-4
4	*ST*1-2	*MH2B*-4	*IS*-4	*IS*-4

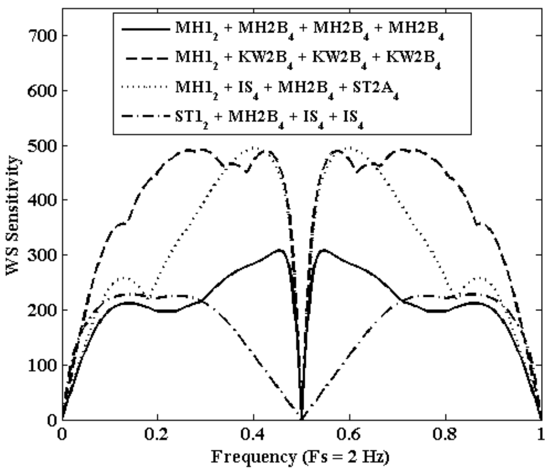


Fig. 5. Worst-case phase-sensitivities of the lower branch (Fig. 1) of a 14th order ALP HT realized as a cascade of different sets of special 2nd and 4th-order allpass sections.

It is seen in Fig. 5, that the proper choice of the individual special allpass sections realizing each TF pole can result in two time smaller WS phase-sensitivity for the HT realization as it was the case for the fourth realization given in Table 1. The realization of the ALP HT which consists of one *ST*1-2, one *MH2B*-4 and two *IS*-4 sections follows the recommendations given in Section 3 and as it can be seen is outperforming the other sets.

V. Low Sensitivity Design

Based on the investigations so conducted and the results so observed, we propose the following design steps in order to achieve higher phase difference accuracy of the approximately linear phase Hilbert transformer in a limited wordlength environment:

1. Obtain $H_{HT}(z)$ (1) according to the application requirements by applying the standard design procedure described in Section 2 and determine the corresponding real allpass TFs (4) as a product of special 2nd and special 4th-order allpass TFs terms.

2. Identify where in the unit circle the special allpass sections TF poles are situated and carefully select from well known (from sources like Section 3, [16][17]) or develop new special allpass sections realizing each couple and quadruplet of poles with the lowest sensitivity and verify this by sensitivity studies as these in Figs. 3, 4.

3. (*Optional – in case of very high accuracy requirements.*) It is better to decompose special second-order allpass sections to two real first order allpass sections, for the real TF poles in the zone ($0.5 < |z| < 0.9$). This research was conducted in our previous investigation described in [12] [13] and will be demonstrated in Section 6.

4. Investigate the overall sensitivities in the imaginary branch of the ALP HT for all possible combinations of the selected special allpass sections realizations in order to select the best set as shown in Section 4.

5. Verify the selection by simulating the structure in a limited word-length environment.

VI. Experiments

The accuracies of the ALP HTs realizations given in Table 1 in a limited wordlength environment are compared in Fig.6. CSD code is used for TF coefficients representation and they are quantized to 4 and 5 significant bits. As it can be seen from Fig. 6, it is difficult to maintain a high accuracy in wider frequency range even in the case of quantization to 5 significant bits. Fig. 6b demonstrates how it is important to analyze the possible TF poles positions in order to select the most proper allpass sections for their realizations. The deviation from the phase quadrature in this case is about ±2°. The highest accuracy in a wider frequency range is achieved by the fourth realization given in Table 1 and 5 bits used for TF coefficients representation and can be seen in Fig. 6d. If there is a need of higher accuracy we can follow the recommendation given in the optional design step (described in the design procedure) - to decompose the special second-order allpass section to two real first order allpass sections having the lowest phase-sensitivity for real TF poles in the zone ($0.5 < |z| < 0.9$). The results so obtained are shown in Fig. 7.

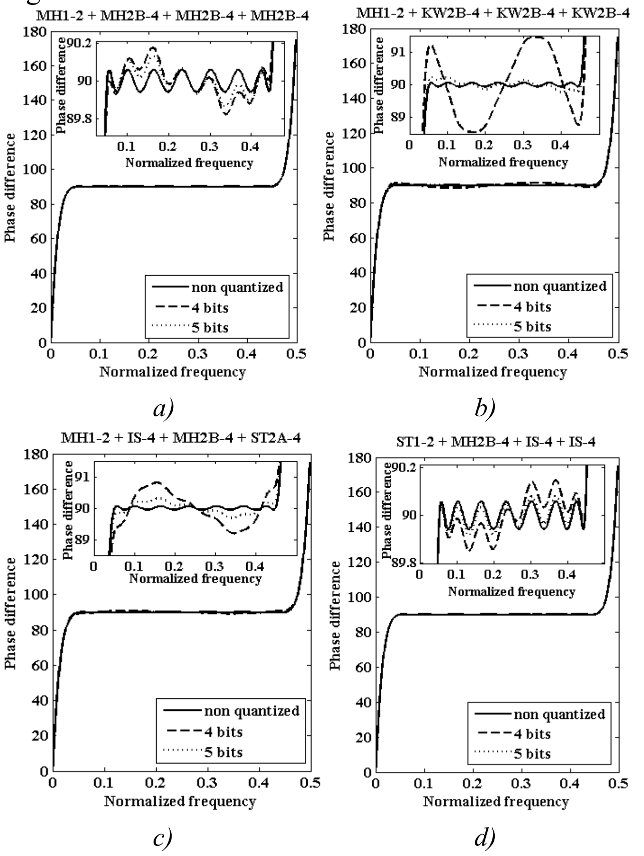


Fig. 6. Wordlength dependence of the accuracy of the ALP HT realized as cascades of different sets of allpass sections.

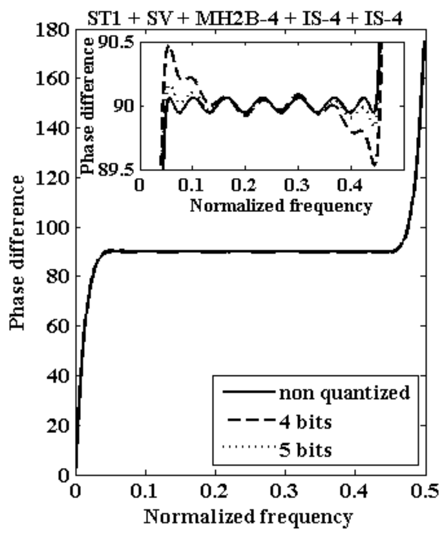


Fig. 7. Accuracy improvement of the phase difference of the ALP HT in a limited worldlength environment when two real first order allpass sections are used instead of the special second order allpass section.

As it can be seen from Fig. 7 the significant improvement of the phase difference accuracy in a wider frequency range closed to the non quantized one in the case of 4 significant bits used for TF coefficients representation is achieved.

VII. Conclusions

It was shown in this work that the worst-case phase-sensitivity minimization of the individual special allpass sections used for the realization of the imaginary branch of the approximately linear phase allpass-based digital Hilbert transformers could considerably improve the accuracy in a limited wordlength environement. Then a proper design procedure also including an optional design step in a case of higher performance requirements was proposed. The effectiveness of the design procedure was experimentally demonstrated. The effect of this is a reduction of the computational load and achievement of a shorter wordlength and lower power consumption for a given accuracy – all very important for realization of portable and mobile communication equipment.

Acknowledgement

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