Three element broadband acoustic array with constant array factor

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In the paper a problem of spatial signals filtration with high selectivity is discussed. To approximate the ideal array factor – Kronecker-δ, approximation method of compressed cosines is used. The approximation with third-degree optimal polynomial is performed. As a result three-element broadband without sidelobes acoustic array with Luneburg lenses is designed. Matlab simulations and comparisons with other acoustic arras are demonstrated.

Триелементна широколентова акустична решетка с постоянен множител на решетката (Петър С. Апостолов, Георги П. Георгиев). *В статията е разгледана високоселективна пространствена филтрация на сигналите. Апроксимирана е делта- функция на Кронекер с нов апроксимационен метод на компресирани косинуси. Апроксимацията се извършва с оптимален полином от трети ред. В резултат на това е синтезирана три-елементна широколентова акустична решетка без странични излъчвания, използваща лещи на Люнеберг. Показани са симулации на Matlab и сравнения с други акустични решетки.*

Introduction

The acoustic arrays are spatial filters. They are used for direct receiving of sound signals to definite direction. They consist of *N* equal uniform set elements (sensors). Basic parameters of acoustic array are:

- general array shape;
- element spacing d normalized against wave length λ;
- element excitation amplitude I_n;
- element excitation phase φ_n;
- pattern of array element.

The gain diagram of acoustic array is defined by the pattern multiplication theorem:

Array pattern = Array element pattern x Array Factor.

The array factor depends of general array shape, element excitation amplitude and element excitation phase. For linear equidistant acoustic array (LEAA) is the function $\sin(nx)/(n\sin x)$. The synthesis procedure reduces to determining of the array factor. The main disadvantage of the acoustic arrays is that they are very narrowbanded. The frequency bandwidth is no more than 10-15% outside the nominal wave length λ.

In the design must be a trade-off between the geometric size and the selectivity of the array. Such a solution is “nested array” [1]. Figure 1 shows array factor of nine – elements nested acoustic array with interelement distance 2.54cm. This array is widebanded: 350 – 7000Hz, but the selectivity is low and inconsistent. So far has not been known a design method for a broadband, narrowbeam LEAA with constant array factor. Subject of research of this paper are LEAA with above mentioned properties.

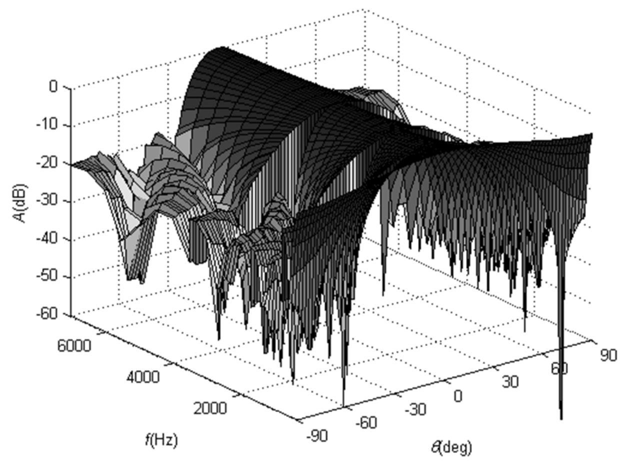

Fig.1. Array factor of nested acoustic array.

Approximation with compressed cosines

There are methods for improving the selectivity of the LEAA. They are created for linear equidistant array antennas design. Their basis is a polynomial approximation of an ideal function. The most popular methods are Taylor's [2], Villeneuve's [3], Schelkunoff's [4], Orchard's [5], Dolph–Chebyshev's [6], Hausdorff's [7] etc. The method of Dolph–Chebyshev and its modification by Riblet [8] have the best properties.

A new polynomial approximation method is used in the paper. The theoretical basis of the method is detailed in [9]. A polynomial approximation of an ideal array factor – Kronecker-δ will be discussed

$$\delta(\theta)=\begin{cases}1, & \theta=0\\ 0, & \theta\neq 0\end{cases};\ \theta\in\left[-\pi/2,\pi/2\right], \tag{1}$$

where θ is the azimuth angle. With the method an optimal 3rd degree polynomial is obtained

$$P_3(\theta)=\sum_{m=1}^{4} b_k \cos\left[(m-1)\varphi(\theta)\right], \tag{2}$$

with coefficients:

$$b_1=b_3=0.5;\ b_2=b_4=0. \tag{3}$$

The argument of the cosine contains a modulating function $\operatorname{erf}(.)$ - Gaussian integral error function with S-shaped graph

$$\varphi(\theta)=(\pi/2)\operatorname{erf}(\beta kd\sin\theta). \tag{4}$$

The modulating function compresses the cosine in the middle of the definition domain, $\beta>0$ is a parameter, which changes the compression density. This effect gives the name of the method: "Approximation with compressed cosines". Figure 2 shows an approximation of the Kronecker-δ with 3rd degree optimal polynomial for two values of the parameter β.

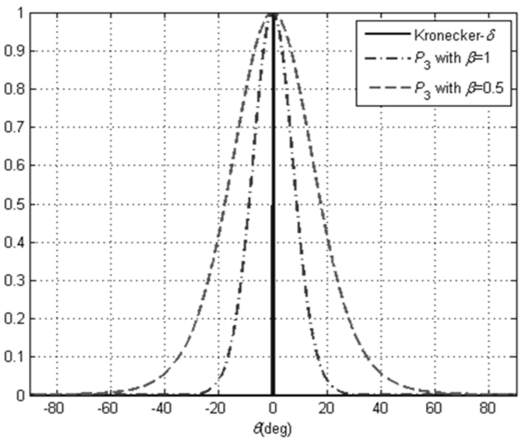

Fig.2. Approximations of Kronecker-δ with third degree polynomials

Figure 3 shows the corresponding array factors in [dB]. It is seen that the array factor is without side lobes and the parameter β changes the width of the mainlobe.

The parameter β is determined from

$$\beta=\frac{\operatorname{erf}^{-1}\left(\frac{1}{\pi}\arccos\left(1-\sqrt{2}\right)\right)}{kd\sin\left(\Delta\theta_{-3dB}\right)}, \tag{5}$$

where $\Delta\theta_{-3dB}$ is the mainlobe width at level $1/\sqrt{2}\approx 0.707$, $\operatorname{erf}^{-1}(.)$ is the inverse integral Gaussian error function.

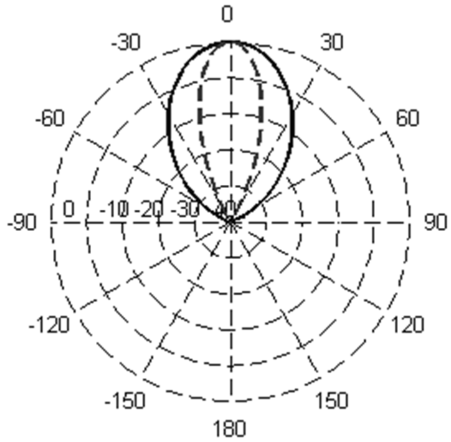

Fig.3. Array factors in [dB]: continues line β=0.5; dashed line β=1.

From the non-zero coefficients of the polynomial the normalized array factor of the acoustic array is determined

$$A(\theta)=1+2\exp\left(j2\varphi(\theta)\right)+\exp\left(j4\varphi(\theta)\right). \tag{6}$$

Design of a three element broadband acoustic array with constant array factor

The task is to realize the three terms of the array factor (6) with technical means. The exciting currents can be easily realized – it is necessary to amplify the signal from the second sensor by 2.

The exponents of the second and the third terms express the phase change of the signals. To be implemented the phase function, it is necessary a technical device to determine the direction of arrival to the sound waves in the azimuth plain. One possible solution is the acoustic Luneburg lens [10], [11]. The Luneburg lens is a sphere with a variable refraction coefficient. Theoretically the refraction coefficient does not depend on the frequency, or the wavelength λ. The lens has the property to focus the parallel rays from all the directions of the azimuth angle θ in points, placed on one semicircle in the azimuth plain as it is shown on figure 4.

In these points microphones are installed. This allows the phase function $\varphi(\theta)$ to be realized with delay lines as it is shown in figure 5.

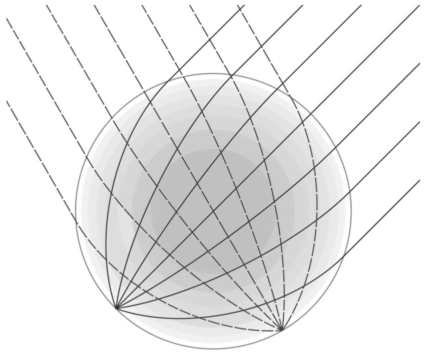

Fig.4. Luneburg lens.

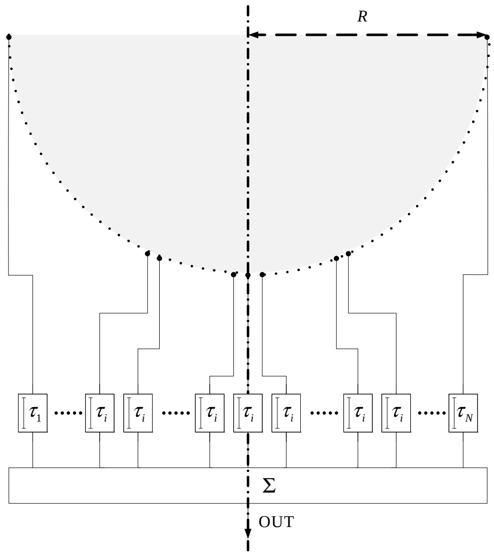

Fig. 5. Lower hemisphere of an acoustic Luneburg lens with delay lines.

The delay times are determined by the modulating function (4). In figure 6 they are shown in graphic form for each of the three lenses, where $\tau_{\max} = \lambda/(2v)$, v is the propagation velocity of the sound in the medium.

Figure 7 shows the structure of a three-element LEAA with the same Luneburg lenses with delay lines. The signal from the second lens is amplified by 2. At the output all the signals are added.

The mainlobe width $\Delta\theta_{-3dB}$ and the time delays τ_L depend on the parameter $\beta > 0$ which can be arbitrarily large. This means that with three lenses, an arbitrary specification can be realized with respect to the width of the mainlobe. The array factor has no sidelobes. The delay time functions for each lens $\tau_L(\theta)$ are formed by two factors: time equalization (phase alignment) for all the directions of arrival, and time delays determined from the modulating function (4) derivative.

These two factors determine two important properties of the acoustic array. The phase alignment makes the array broadband. The derivative of the modulating function is significant as a weighting function, which determines the selectivity of the main lobe. Thus, at a fixed interelement distance, the acoustic array can operate with a constant selectivity of the array factor on an arbitrary nominal frequency or wavelength.

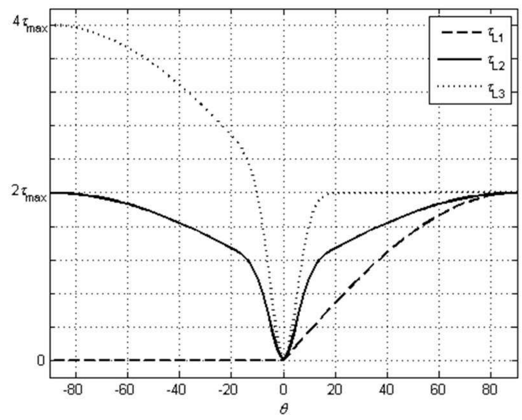

Fig. 6. Time delays functions.

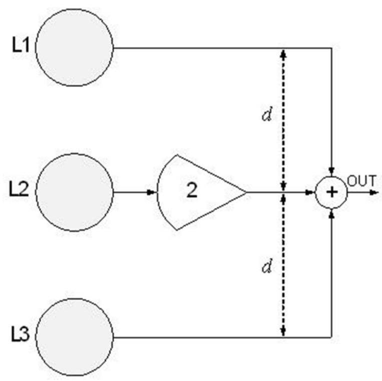

Fig. 7. Scheme of LEAA with 3 Luneburg lenses

The described property is proved by a computer simulation of Matlab for 3 element array with interelement distance 17.1cm and mainlobe width $\Delta\theta_{-3dB} = 10^0$. For frequencies 300, 1000 and 3500Hz, 91×3 sinusoids were created with a sampling rate of 192 kHz. The sinusoids are phased so as, to simulate reception of a signals at the three points of the sensors, from 91 spatial directions of the azimuth angle $\theta \in [-\pi/2, \pi/2]$. Figure 8 shows the results of the simulation. For the different frequencies the beam-width is constant. For 3500Hz the discretization noise is at level -30dB.

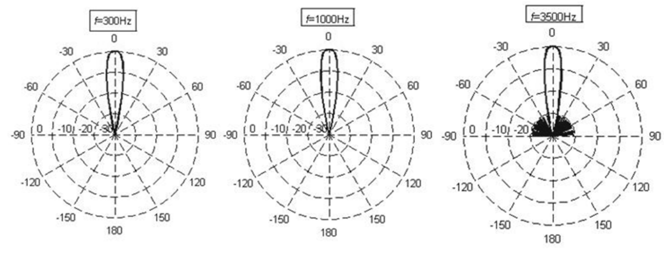

Fig. 8. Matlab simulation.

Discussion

In Fig. 9, the array factors of four acoustic arrays are compared with the following specifications:

- three element uniform array with interelement distance $d = \lambda/2$;
- three element binomial array with interelement distance $d = \lambda/2$;
- fifteen element Dolph-Chebyshev array with interelement distance $d = \lambda/2$, beamwidth $\Delta\theta_{-3dB} = 9.6^0$ and side lobes level -40 dB;
- three element compressed cosines array with interelement distance $d = \lambda/2$ and beamwidth $\Delta\theta_{-3dB} = 9.6^0$.

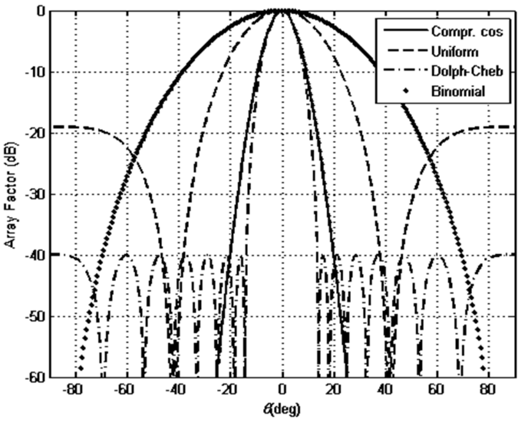

Fig. 9. Array factors comparison

It is seen that the acoustic array with compressed cosines has better selectivity than uniform and binomial arrays; wider main lobe, but no side lobes as compared with the Dolph-Chebyshev array.

In the paper, it is assumed that the Luneburg lens has unit gain. For aperture efficiency 0.6, this condition is satisfied when the diameter of the lens is

$$D \approx 0.411\lambda . \quad (7)$$

For larger values of the diameter, it is necessary to take the gain of the lens into account and the radiation of the array will be defined by the pattern multiplication theorem.

The accuracy of the array factor beamwidth depends on the number of sensors *N* (Fig. 5). For the above demonstrated simulation with 91 sensors (Fig. 8), the narrowest accurate beamwidth is four degrees.

The described properties of the LEAA imply the use of digital signal processing. This allows the acoustic array to operate in real time on one or more nominal frequencies, with equal array factor selectivity, and also steer the main beam.

The signal from the second lens must be multiplied exactly by 2; otherwise, side lobes appear. Exact multiplication by 2 and programmable time delays with digital signal processing can be easily realized.

Conclusion

With the method of compressed cosines an optimal 3^{rd} degree polynomial is obtained. A technical solution to design LEAA with 3 acoustic Luneburg lenses is proposed. As a result a broadband without sidelobes LEAA, with constant array factor is designed.

Acknowledgements

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