Analysis of a current source supplied inverter

Peter T. Goranov

The article describes the performance of parallel current fed inverter, supplied with ideal current source. The analytical expressions are derived, basic characteristics and parameters are shown with resistive and resonant load. The studies relate to the thyristor and transistor implementation, although the drawings show thyristor circuits. To assess the ability of this version the current fed inverter is initially investigated with resistive load, then are derived analytical relationships for complex load - a parallel resonant circuit. Equations are obtained, showing the influence of load or frequency variation on voltages in the scheme and parameters defining the reliable operation of the researched inverter. The results obtained are compared with the known dependencies of the standard scheme of the current fed inverter. Studies have been confirmed with simulation results. Mode of operation is verified experimentally. Recommendations are made for the design of the inverter.

Анализ на инвертор на ток, захранван с източник на ток (Петър Горанов). *В статията се анализира работата на паралелен инвертор на ток, захранван с идеален източник на ток. Изведени са аналитични изрази, показани са основни характеристики и параметри при активен и комплексен товар. Изследванията се отнасят за тиристорни и транзисторни варианти, въпреки че на фигурите са показани тиристорни схеми. За оценка на възможностите на този специфичен вариант на инвертора на ток в началото е изследвана работата при активен товар, след което са изведени аналитични зависимости за комплексен товар – паралелен трептящ кръг. Получени са уравнения, показващи влиянието на изменението на товара или работната честота върху напреженията в схемата и параметрите, определящи работоспособността на изследваната схема. Получените резултати са сравнени с известните зависимости за стандартната схема на инвертор на ток. Изследванията са потвърдени със симулационни резултати. Режимът на работа е проверен експериментално. Направени са препоръки за проектирането на инвертора.*

Introduction

The standard inverter scheme is supplied by a voltage source and includes an inductor with high inductance that ensures constant value of the supplied current (Figure 1). The inverter circuit is examined in detail and analyzed in the literature [1], [2], [3].

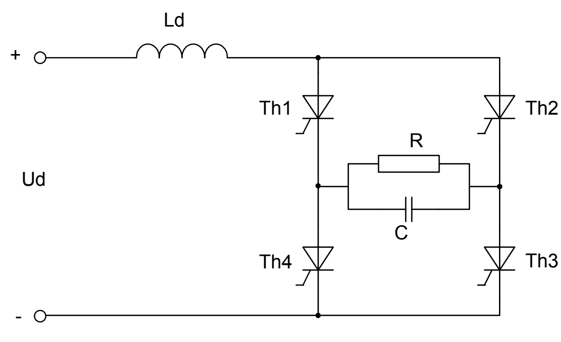

Fig.1. Standard inverter circuit.

There are situations when the power supply source is with current source characteristics. Then the current fed inverter looks like the scheme shown in Figure 2. Such type of a power source could be easily realized using switched mode DC-to-DC converters or photovoltaic panels.

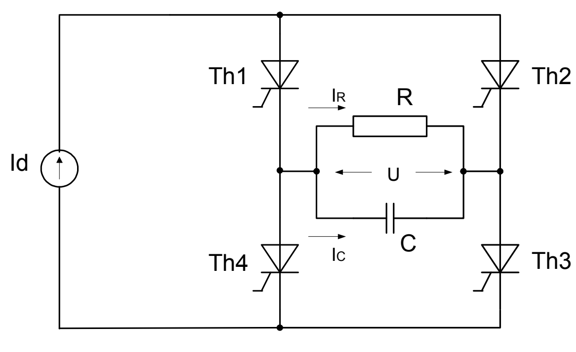

Fig.2. Current fed inverter current source supplied.

When the standard current fed inverter is supplied by voltage source input current is almost perfectly smooth, but may have an arbitrary (and unlimited) constant value. Current source supplied current fed inverter has an invariable input current and independent of load variations or operation modes.

A variant of real implemented circuit is shown in Figure 3. The power DC-to-DC converter composed of transistor T, diode D and choke Ld, is in mode of current stabilizer. Usually the load for such inverters is a resonant circuit – the choice is for a parallel one.

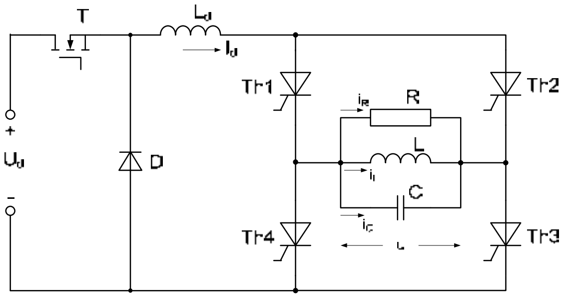

Fig.3. An option of current source supplying.

The purpose of the analyses in the paper is an investigation the influence of load variations or operating frequency on the inverter output voltage and the parameters defining the proper operating of a parallel inverter supplied by current source.

The research and results are related also to transistor circuit.

A detailed analysis of the investigated inverter with resistive load is made in [4], so here are some more important results. It is proposed analysis under real load – resonant circuit.

Analysis – resistive load

A. Output voltage with resistive load

The following system of equations describes the bridge circuit of the current fed inverter (Figure 2)

(1) $$I_d = i_R + i_C \quad ; \qquad i_R = \frac{u}{R} \quad ; \quad i_C = C\frac{du}{dt} \quad ,$$

where u is a load voltage, i_R – resistive load current, i_C – capacitive current, I_d – input current, R – load resistance, C – load capacitor.

The differential equation, related to every half-period, and it's solution are described in the literature [5]

(2) $$RC\frac{du}{dt} + u = R.I_d \quad ,$$

(3) $$u = R.I_d(1 - \frac{2}{1+e^{-\frac{B}{2}}}e^{-\frac{B}{T}t}) \quad ,$$

where $B=1/(fRC)$ is a load coefficient.

The effective (*rms)* value of the output voltage is obtained using expression (3) when solving the equation

(4) $$U = \sqrt{\frac{2}{T}\int_0^{T/2} u(t)^2\,dt} \quad ,$$

where T is the period of operating frequency.

The result for the *rms* value of the output voltage is

(5) $$U = R.I_d\sqrt{(1 - \frac{4}{B}\frac{1-e^{-\frac{B}{2}}}{1+e^{-\frac{B}{2}}})}$$

or

(6) $$U = R.I_d\sqrt{(1 - \frac{4}{B}th\frac{B}{4})} \quad .$$

To obtain the generalized result the equation (5) is converted in relative units using the expression

(7) $$\overline{U} = \frac{U}{U_{oc\max}} = \frac{4}{B}\sqrt{(1 - \frac{4}{B}\frac{1-e^{-\frac{B}{2}}}{1+e^{-\frac{B}{2}}})} \quad ,$$

where

$$U_{oc\max} = \frac{I_d}{C}\frac{T}{4}$$

is the maximum of output voltage at no load.

The dependency of the *rms* value in relative units $\bar{U}$ on the coefficient B is given in Figure 4 with solid line. For comparison the same dependency for the scheme from Figure 1 is shown with a dotted line.

In contrast to the unlimited increase of the output voltage under load open circuit *(R→∞)* for the inverter supplied by voltage source the examined scheme has limited variations of load voltage. This allows to control the output voltage varying the input current I_d. It could be seen that the relatively small change of the current I_d provides a wide voltage load variations, which produces well controll of the output voltage.

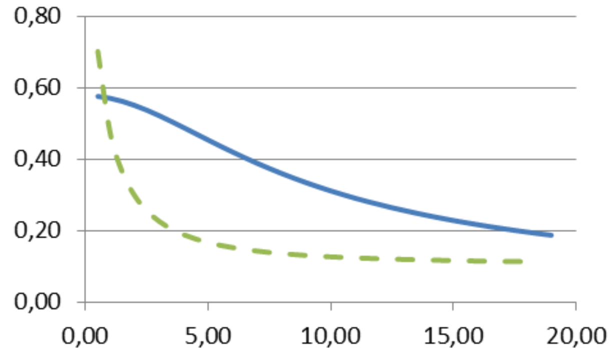

Fig.4. Output voltage versus the coefficient B at resistive load.

B. Output power

Output power is calculated from

(8) $$P = \frac{U^2}{R}$$

or in relative units

(9) $$\overline{P} = \frac{P}{U_{oc\max}.I_d} = \frac{4}{B}\left(1 - \frac{4}{B}\frac{1-e^{-\frac{B}{2}}}{1+e^{-\frac{B}{2}}}\right)$$.

Graphically the expression (9) is presented on Figure 5. The power dependency of *B*-coefficient for current source supplied current fed inverter shows wide maximum. This defines the clear criteria for *B*-coefficient choice and makes easy circuit design. The real advisable range of *B*-variation is much larger – from *B*=5,5 to *B*=7,5.

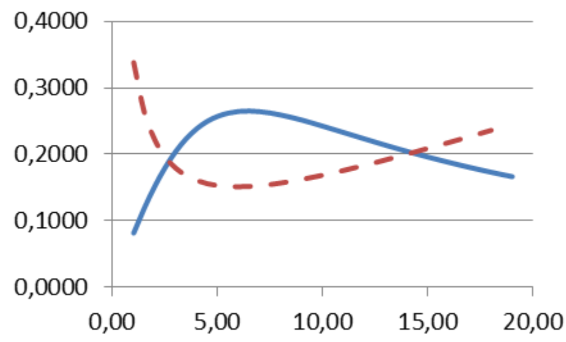

Fig.5. Output Power versus the coefficient B.

Figure 5 shows also the typical minimum of the output power and the corresponding minimum of the input current (the dotted line) in the standard supply by voltage source.

Analysis – real load

A. Basic equations

The real circuit mentioned above (Figure 3) is investigated. Analysis is based on the equivalent diagram corresponding to processes in the inverter shown in Figure 6.

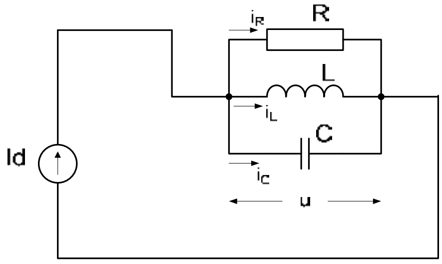

Fig.6. Equivalent circuit.

In accordance with the equivalent circuit, the following equations are written

(10) $$I_d = i_R + i_C + i_L$$,

(11) $$I_d = L.C.\frac{d^2 i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L$$,

where

(12) $$i_R = \frac{U}{R} \quad , \quad i_C = C\frac{du}{dt} \quad , \quad u = L\frac{di_L}{dt}$$.

Solution to the differential equation (11) looks like

(13) $$i_L = e^{-\frac{t}{\tau}}(a.\sin \omega_0 t + b.\cos \omega_0 t) + I_d$$,

from where the expression for load voltage

(14) $$u = \omega_0 L.e^{-\frac{t}{\tau}}[a.(\cos \omega_0 t - \frac{1}{\omega_0 \tau}\sin \omega_0 t) - b.(\sin \omega_0 t + \frac{1}{\omega_0 \tau}\cos \omega_0 t)]$$.

In the analysis are used parameters as follows:

- ideal resonant frequency

(15) $$\omega_r = \sqrt{\frac{1}{L.C}}$$;

- load resonant frequency

(16) $$\omega_0 = \sqrt{\omega_r^2 - \frac{1}{\tau^2}}$$;

- load time-constant

(17) $\tau = 2RC$;

- output (operating) frequency $\omega = \nu.\omega_0$, where

(18) $\nu = \dfrac{\omega}{\omega_0}$

is a frequency coefficient and for thyristor circuits must be $\nu>1$, while for transistor implementation $\nu=1$ is recommended;

- load coefficient B and related expressions are converted to a more general form, based on the equations (15)÷(18)

(19) $B = \dfrac{1}{f.R.C} = \dfrac{2\pi}{\omega.R.C} = \dfrac{4\pi}{\nu.\omega_0.\tau}$

and

(20) $\dfrac{1}{\omega_0.\tau} = \dfrac{B.\nu}{4\pi}$.

B. Voltage and current equations

Solving the equations for the current through the inductor $i_L(t)$ and for the load voltage $u(t)$ is based on the determination of the unknown constants *a* and *b* in initial conditions

(21) $i_L(0) = I_o \quad , \quad u(0) = -U_0$.

After substituting in equation (13) and (14) is obtained

(22) $a = \dfrac{1}{\omega_0\tau}(I_0 - I_d - \dfrac{U_0.\tau}{L}) \quad , \quad b = I_0 - I_d$.

Initial values for current I_0 and voltage U_0 are determined under assumption that at the end of half-period – $t=\pi/\omega$, fulfills the conditions

(23) $i_L(\dfrac{\pi}{\omega}) = -I_o \quad , \quad u(\dfrac{\pi}{\omega}) = U_0$.

After substituting and solving the equations, the following expressions are obtained

- for the current initial condition I_0

(24) $I_o = -(2.k_1 - 1).I_d$,

where

$$k_1 = \frac{1}{1 - e^{-\frac{\pi}{\nu}.\frac{1}{\omega_0\tau}}.\left(\dfrac{\dfrac{1}{\omega_0\tau}.\sin\dfrac{\pi}{\nu} + \cos\dfrac{\pi}{\nu} + e^{-\frac{\pi}{\nu}.\frac{1}{\omega_0\tau}}}{e^{-\frac{\pi}{\nu}.\frac{1}{\omega_0\tau}}.(\dfrac{1}{\omega_0\tau}.\sin\dfrac{\pi}{\nu} - \cos\dfrac{\pi}{\nu}) - 1}\right)}$$

or

(25) $k_1 = \dfrac{1 + e^{-\frac{B}{4}}.\left(\cos\dfrac{\pi}{\nu} - \dfrac{B.\nu}{4\pi}\sin\dfrac{\pi}{\nu}\right)}{1 + e^{-\frac{B}{4}}.(2\cos\dfrac{\pi}{\nu} + e^{-\frac{B}{4}})}$.

- for the voltage initial condition U_0

(26) $U_o = \omega_0 L.I_d \dfrac{2k_1.e^{-\frac{B}{4}}.\left(1 + (\dfrac{B.\nu}{4\pi})^2\right).\sin\dfrac{\pi}{\nu}}{1 + e^{-\frac{B}{4}}.\left(\cos\dfrac{\pi}{\nu} - \dfrac{B.\nu}{4\pi}\sin\dfrac{\pi}{\nu}\right)}$.

- inductive current equation

(27)
$$i_L(t) = I_d + (I_0 - I_d).e^{-\frac{t}{\tau}}.(\frac{B.\nu}{4\pi}.\sin\omega_0 t + \cos\omega_0 t) - \\ - \frac{U_0}{\omega_0 L}.e^{-\frac{t}{\tau}}.\sin\omega_0 t$$

- load voltage equation

(28)
$$u(t) = -\omega_0 L.(I_0 - I_d).\left(1 + (\frac{B.\nu}{4\pi})^2\right).e^{-\frac{t}{\tau}}.\sin\omega_0 t - \\ - U_0.e^{-\frac{t}{\tau}}.(\cos\omega_0 t - \frac{B.\nu}{4\pi}.\sin\omega_0 t)$$

Summarized form of the obtained expressions for the current $i_L(t)$ and the voltage $u(t)$ are prepared by converting them into relative units. Due to this the survey of the inverter only requires knowledge of the coefficients *B* and *ν*. For this purpose the correlations are used

(29) $i^*(t) = \dfrac{i(t)}{I_d} \quad , \quad u^*(t) = \dfrac{u(t)}{\omega L.I_d} = \dfrac{u(t)}{\nu.\omega_0 L.I_d}$.

By changing the current variable – time *t*, with angle variable – $\theta=\omega t$ or $t=\theta/\omega$, expression for the current through the inductance is brought in the form

(30)
$$i_L^*(\theta) = 1 - 2k_1.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\left(\frac{B\nu}{4\pi}.\sin\frac{\theta}{\nu} + \cos\frac{\theta}{\nu}\right) - \\ - k_2.\nu.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\sin\frac{\theta}{\nu}$$,

where

$$(31)\qquad k_2 = \frac{2k_1}{\nu}.\frac{e^{-\frac{B}{4}}.\left[1+\left(\frac{B\nu}{4\pi}\right)^2\right].\sin\frac{\pi}{\nu}}{1+e^{-\frac{B}{4}}\left(\cos\frac{\pi}{\nu}-\frac{B\nu}{4\pi}.\sin\frac{\pi}{\nu}\right)} \quad .$$

The output voltage in relative units presents by

$$(32)\qquad \begin{aligned} u^*(\theta) = {} & \frac{2k_1}{\nu}.\left[1+\left(\frac{B\nu}{4\pi}\right)^2\right].e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\sin\frac{\theta}{\nu} - \\ & - k_2.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.(\cos\frac{\theta}{\nu}-\frac{B\nu}{4\pi}\sin\frac{\theta}{\nu}) \end{aligned} \quad .$$

Using the substitution

$$(33)\qquad \begin{aligned} \frac{1}{tg\alpha} = {} & \frac{\frac{2k_1}{\nu}.\left[1+\left(\frac{B\nu}{4\pi}\right)^2\right]}{k_2} + \frac{B\nu}{4\pi} = \\ & = \frac{1+e^{-\frac{B}{4}}.\cos(\frac{\pi}{\nu})}{e^{-\frac{B}{4}}.\sin(\frac{\pi}{\nu})} \end{aligned} \quad ,$$

the equation (32) is converted to

$$(34)\qquad u^*(\theta) = \frac{k_2}{tg\alpha}.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\sin\frac{\theta}{\nu} - k_2.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\cos\frac{\theta}{\nu} \quad .$$

Based on simple geometrical correlations more convenient expression (35) is obtained

$$(35)\qquad u^*(\theta) = \frac{k_2}{\sin\alpha}.e^{-\frac{B\nu}{4\pi}.\frac{\theta}{\nu}}.\sin(\frac{\theta}{\nu}-\alpha) \quad .$$

Due to expressions derived the circuit could be analyzed. It is very easy to estimate the influence of varying operating conditions – load, frequency, over output voltage. The shape of output voltage for different values of load coefficient B is shown on fig.7 at *ν=1,1*. Figure 8 shows the influence of different output frequencies.

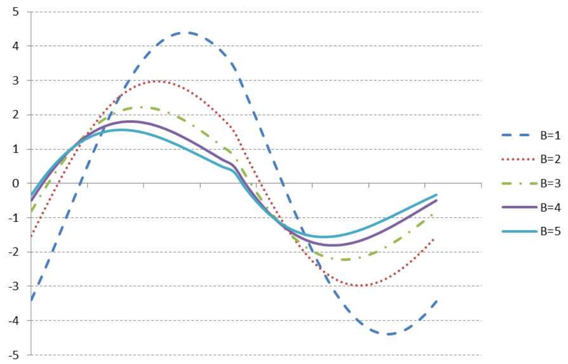

Fig.7. Voltage shape versus load coefficient B.

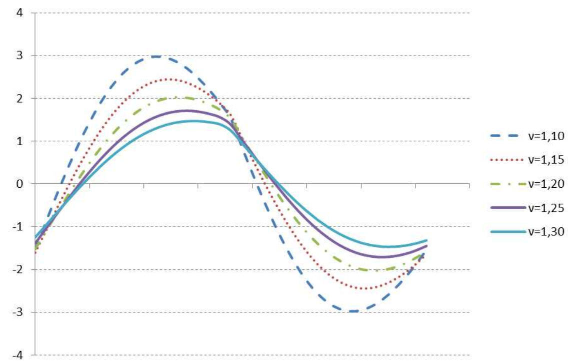

Fig.8. Output voltage shape versus frequency coefficient ν at B=const.

C. Effective value of the output voltage

The effective value of the output voltage is obtained on the basis of expression (35) by solving the equation

$$(36)\qquad U = \sqrt{\frac{1}{\pi}\int_0^{\pi} u^2(\theta)d\theta} \quad .$$

A complicated formulae (37) is obtained. It allows studying the dependency between *rms* value of output voltage and load coefficient *B* and frequency coefficient *ν*. These relationships are very multilateral and based on dependence (37) can be analyzed in each case.

$$(37)\qquad U^*_{eff} = \frac{k_2}{\sin\alpha}\sqrt{\frac{1}{B}.\left\{1-e^{-\frac{B}{2}}+\frac{\left(\frac{B\nu}{4\pi}\right)^2}{1+\left(\frac{B\nu}{4\pi}\right)^2}.\left[e^{-B}.\left(\cos(2\frac{\pi}{\nu}-2\alpha)-\frac{1}{\frac{B\nu}{4\pi}}\sin(2\frac{\pi}{\nu}-2\alpha)\right)-\cos 2\alpha-\frac{1}{\frac{B\nu}{4\pi}}\sin 2\alpha\right]\right\}}$$

Some of them have been plotted in Figure 9 and Figure 10.

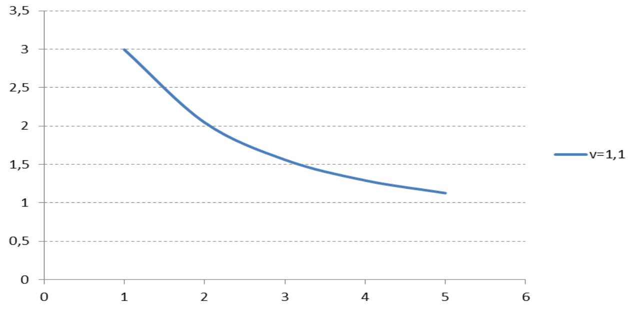

Fig.9. Output voltage U_{eff} versus load coefficient B at v=1,1.

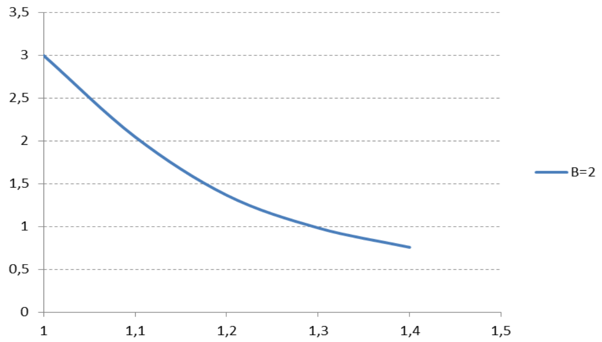

Fig.10. Output voltage Ueff versus frequency coefficient v at B=2.

D. Circuit turn-off time

The circuit turn off time in a thyristor circuit t_{qs} , respectively turn off angle θ_{qs}, has to be large enough to assure the proper inverter operation. It is obtained from the equation (35) by executing the condition

(38) $u(\theta_{qs}) = 0$,

where $\theta_{qs} = \omega . t_{qs}$.

The turn off angle θ_{qs} is a solution of the equation

(39) $\frac{k_2}{\sin\alpha} . e^{-\frac{B\nu}{4\pi} . \frac{\theta_{qs}}{\nu}} . \sin(\frac{\theta_{qs}}{\nu} - \alpha) = 0$,

which is

(40) $\theta_{qs} = \nu . \alpha$.

Of course, the angle α is determined by the equation (33).

The dependency of the turn-off angle (in radians) on circuit coefficients B and ν is plotted in Figure 11 and Figure 12.

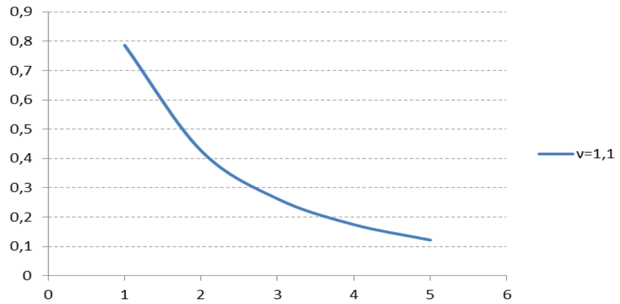

Fig.11. Turn-off angle versus load coefficient B at v=1,1.

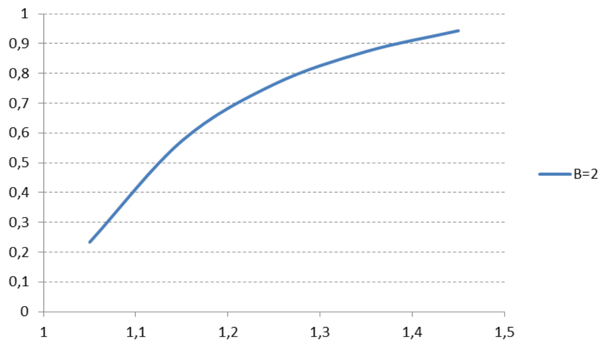

Fig.12. Turn-off angle frequency coefficient v at B=2.

E. Simulation results

To confirm the obtained equations and dependencies the current source supplied current fed inverter, transistor implementation, is simulated (Figure 13). Circuit parameters are as follows: load inductance – 31,6μH, load capacitance – *2μF*, load resonant frequency approximately – ω_0*=20kHz*, supply current – *Id=10A*. Load resistance has three different values *R=25Ω*, *12,5Ω*, *6,25Ω*, respectively the value of load coefficient is *B=1, 2, 4*. Operating frequency is *ω=22kHz* and correspondingly the frequency coefficient is *ν=1,1*.

With these rates of coefficients B and ν are plotted calculated results for output voltage in Figure 7. According to mentioned parameter values the relative voltage ratio is

(41) $\omega L . I_d = 2\pi . 22.10^3 . 31.10^{-6} . 10 = 42,85132$.

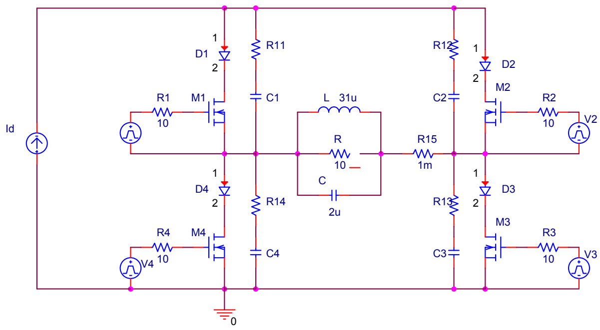

Fig.13. Simulated circuit.

Figure 14 shows the simulation results for output voltage, which confirm obtained expressions and calculations.

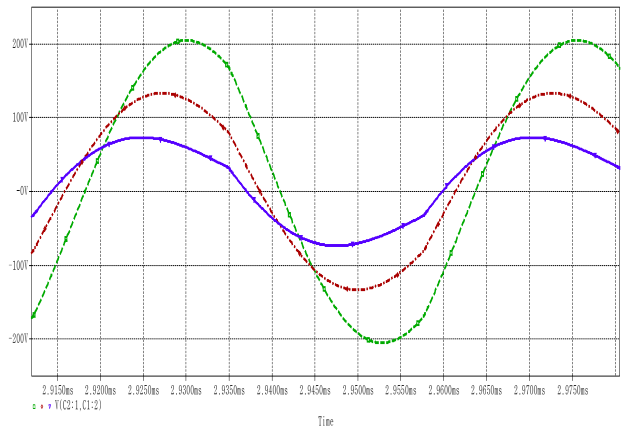

Fig.14. Simulation results.

Conclusions

As a result of the analysis of the inverter powered by ideal current source can draw the following conclusions.

Relations obtained allow to investigate different modes of operation, to assess the specific working conditions and to design the inverter with given load parameters.

An important difference compared to standard current fed inverter: the output voltage has highest value when operating frequency is equal to load resonant frequency, which is only possible with transistor circuit.

It is proved the stable inverter performance under extreme load values – from load short circuit to pure capacitive load. Infinite increase of load voltage is not observed. Under the short circuit currents in the scheme are defined from the input current and have preliminary determined value.

Main method to control the output voltage is by variation of the supply current. Within certain limits it is possible to make this modifying the operating frequency.

The obtained dependencies allow examine the thyristor and transistor circuits.

Analysis with resistive load offers more general assessment of the capabilities of the inverter fed by ideal current source.

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Assoc. Prof. Petar T. Goranov PhD *is professor at Power Electronics Department of TU-Sofia, he received the master degree from VMEI- Sofia in 1973, where he is working on power electronic converters and their control systems.*

tel.:0888 953 048   e-mail: pgoranov@ecad.tu-sofia.bg

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