From electrica to invariant automatica
(Or how to use the knowledge about Theory of electricity for enter into Theory of invariant automatic control).

Part two: Electromechanical dualism. Universality of energetic equations.

Milan Stankov

От електрика към инвариантна автоматика (или как да използваме познанията си по „Теоретична електротехника" за да вникнем в управлението на технологичните процеси), Втора част. Електромеханична двойственост. Универсалност на енергийните равенства (Милан Станков).

A. Electromechanical dualism.

1. Movement of a material point in resistive medium.

Axiom A2.1. The full energy E_m that a material point with constant mass m consumes from its operating source presents the scalar product:

(2.1) $E_m = \int\limits_0^t VF dt = VFt = St$, where

V is effective value of the velocity v of the point [m/s], F – effective value of the – resistive force f of the medium [N] in that moves the point, S – consumed by the point full power.

Axiom 2.2. The resistive force f presents the vector sum:

(2.2) $f = f_v + f_a + f_s$, where:

(2.3) $f_v = \eta v$ is the resistive force [N] of the friction in the medium,

(2.4) η – the coefficient of the friction [Ns/m] or [Ps.m],

(2.5) $f_a = m\dfrac{dv}{dt}$ - mass inertia force [N] of the point,

(2.6) m – mass of the point [kg],

(2.7) $f_s = \dfrac{1}{k}\int\limits_0^t v dt$ - resistive elastic force [N] of the medium,

(2.8) $k = \dfrac{1}{c}$ - coefficient of antielasticity (rigidness) of the medium [m/N],

(2.9) c – coefficient of elasticity of the medium [N/m], from that follows the trivial proved:

Theorem T2.1. The resistive force from equation (2.2) will have the description:

(2.10) $f = \eta v + m\dfrac{dv}{dt} + \dfrac{1}{k}\int\limits_0^t v dt$.

Axiom 2.3. Velocity of the point v is repeatedly lower than the velocity c_S of the light.

Axiom 2.4. Time t is measured in absolute calendar mode from that follows the trivial proved:

Theorem T2.2. Movement of the point is described in the time t in immovable three-dimensional co-ordinate system (0, x, y, z) or in movable three-dimensional co-ordinate system (0′, $x′, y′, z′$), that moves along the axis x of the immovable system with the velocity v_0 = const.

Consequence C2.1. (without proof). Energetic equations described in both systems are invariant about the transformation ns of Galilee:

(2.11) $x′=x-v_0t, y′=y, z′=z$,

that presents the **principle of the relativity of Galilee.**

Theorem T2.3. Between electrical energetic equation (1.2) from the first part of this article and mechanical energetic equation (2.1) exists a class of

functions Φ, transforming dual the first equation in the second.

Proof: On the diagram of figure 2.1 are described like composite functions the consecutive analytical transformations leading to the equations (1.2) and (2.1). The algebraic structures formed from these transformations are known under the name **functors.** Electrical functor is dual similar of the mechanical, that is in evidence in the comparison between mechanical equation (2.1) and electrical (1.2), mechanical equation (2.2) and electrical (1.1) and the elements of mechanical vector sum (2.10) between electrical vector sum (1.1).
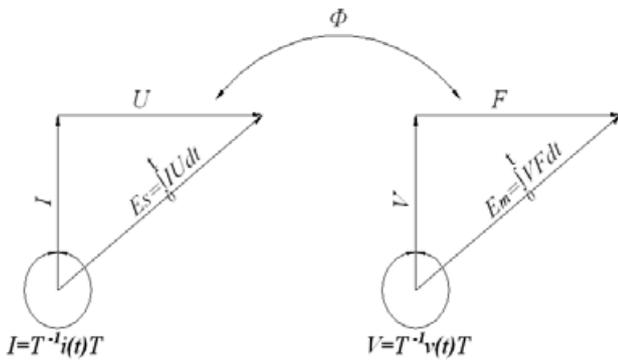

Figure 2.1. Dual similar functions.

The algebraic dual similarity is full. That is described in the comparative table on the figure 2.2 (see below). And it means that the class of functions Φ exists.

SYSTEM			
ELECTRICAL		MECHANICAL	
Value	Symbol	Symbol	Value
Current	i	v	Velocity
Voltage	u	f	Force
Resistance	R	η	Friction
Inductivity	L	m	Mass
Capacity	C	k	Anti-elasticity

Figure 2.2. Dual electromechanical similarity.

On the basis of this similarity is composed the international measuring system SI (see its basic and electrical measures on figures below).

BASIC MEASURES ACCORDING SI				
	Value	Symbol	Name	Measure
1	Length	m	meter	m
2	Mass	kg	kilogram	kg
3	Time	s	second	s
4	Current	A	Ampere	A

Figure 2.3. Basic measures according SI

ELECTRICAL MEASURES ACCORDING SI				
No.	Value	Symbol	Name	Measure
1	Current	A	Ampere	A
2	Voltage	V	Volt	$m^2 kg / As^2$
3	Resistance	R	Ohm	$m^2 kg / A^2 s^3$
4	Inductivity	L	Henry	$m^2 kg / A^2 s^2$
5	Capacity	C	Farad	$A^2 s^4 / m^2 kg$

Figure 2.4. Electrical measures according SI

All electrical measures are according SI composed by the basic measures (see above figure 2.4).

The value *Ampere* has also a mechanic measure. According SI *Ampere* (*A*) is an unchangeable current that passing through two interminable wires in vacuum with ignorable round section and to a distance one meter between them, will provoke between same wires a force 2.10^{-7} N/m.

Theorem is proved.

Theorem T2.4. Mechanical energetic equation (2.1) is dual similar of all possible invariant physical energetic equations whose action and contra-action, respectively – productivity and quality are function of the absolute calendar time t (see axiom A2.4).

Proof: System SI contains mechanical measures for all physical values.

2. Energetic dual technologic equipments and process.

2.1. Thermal (calorific) exchanger

Energy E_T that is radiated by a thermal exchanger (radiator, air-heater etc.) is defined by the equation:

$$(2.11) \quad E_T = \int_0^t c_T (T_1 - T_2) Q dt , \text{ where}$$

c_T is coefficient of the thermal exchange [Ws/kg. ^{0}C],
T_1 – temperature on the input of the exchanger [^{0}C],
T_2 – temperature on the output of the exchanger [^{0}C],
Q – mass flow of the thermal carrier [kg/s].

2.2 Hydraulic turbine.

Energy E_H that the hydraulic stream transfers to the blades to a hydraulic turbine is defined by the equation:

(2.12) $E_H = \int\limits_0^t \gamma g H Q dt$,

where γ is density of the water [kg/m^3], g – Earth acceleration [m/s^2], H – geodesic high of the hydraulic column [m], Q – hydraulic flow [m^3/s].

B. Disturbance of the invariability. Automatic control.

Definition D2.1. System described on figure 2.5 by that the output value x is compared through reverse connection (loop) with the input set point x^0 to be achieved equality with him, is named **closed (closed controlled** or **automatic controlled)** system.

Theorem T2.5. Linear change of impedance of the system or the energy E_S toward him saves in contra-phase consumed by the object O energy E_{SO} in relation to energy E_S.

Proof: Linear change of the energetic vectors E_S and E_{SO} save orthogonal active, reactive and deformed components one in relation to other. And that is possible when the three components of the impedance of the system - active, inductive and capacitive changes linear, saving its correlations. In this way two energies E_S and E_{SO} according theorem T1.2 are in contra-phase one in relation to other and the system pass from a stable state before the change to other after him.

Theorem T2.6. By disturbance of invariability of the system through linear change of impedance of the system or the energy E_S toward him it is necessary and sufficiently the system to be closed controlled (automatic controlled) by the scheme on figure 2.5.

Proof: Signal x on the output of the controlled object O is feed to set point device to *PID* controller $P+I+D$ that compares it with the signal x^0 of the programmed set point trough subtraction of x^0 from x. If the difference $\Delta x^0 = x^0 - x$ is zero the dynamic equilibrium of the system is not changed.

The *PID* controller $P+I+D$ don't changes its output signal. Object O receives the energy $E_S = E_S(x^0)$ and characteristic signal x of the output product of object O is equal to x^0.

If, however, it is not just so to the input of *PID* controller $P+I+D$ will be generated the signal:

(2.13) $y^0 = x^0 \pm \Delta x^0$,

that will be transformed according *PID* law in the signal:

(2.14) $z^0 = k_P y^0 + T_D \dfrac{dy^0}{dt} + \dfrac{1}{T_I} \int\limits_0^t y^0 dt$

where $y^0 = x^0 \mp \Delta x^0$.

Signal z^0 will forces source *ES* to change energetic flow E_S just so that it to be present like the contra-function:

(2.15) $E_S = E_S(z^0 \rightarrow x^0)$

and by this way the changed signal $x^0 \pm \Delta x^0$ to return its primary value x^0. There pass the transition:

(2.16) $\lim(x^0 \pm \Delta x^0) \rightarrow x^0, t \rightarrow \infty$

Transition (2.16) is possible by linear changes of the energetic flow E_S or of impedance multitude of the object O:
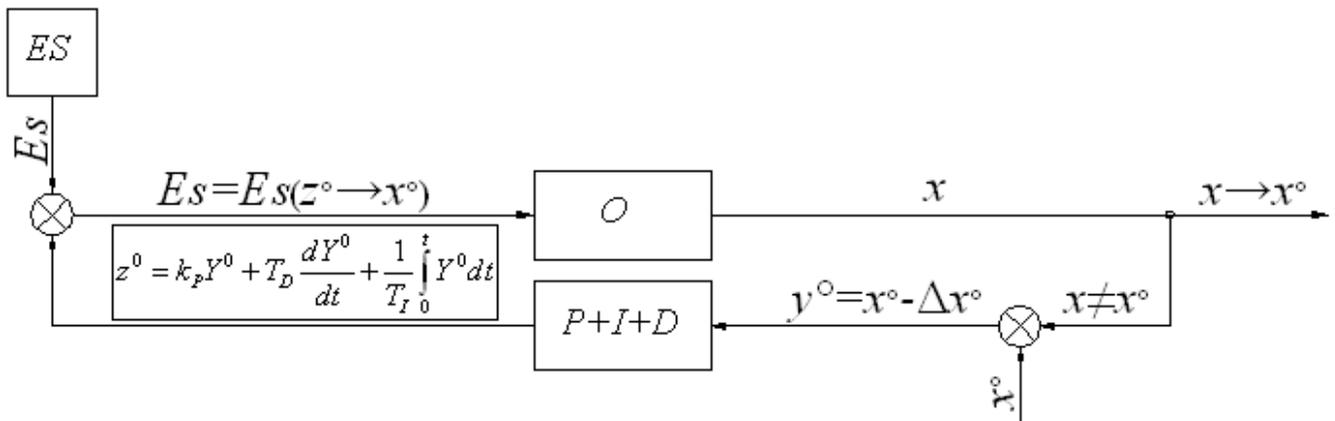

Figure 2.5. Closed controlled system.

(2.17) $Z_{HO} : \{k_{PO}, T_{DO}, \frac{1}{T_{IO}}\}$.

Then according theorem T2.5 the system pass from one stable state before the change to other after him saving the value x^0 on the output of the object O.

Theorem is proved.

Theorem T2.7. (whitout proof). By non linear change of impedance of the system or the energy E_S toward him the process must be stop. A diagnostic of the state of system must be made and after that must be made the necessary tunings for a new linearization.

C. Stability of the system.

Definition D2.2. State by that the output signal x of the system obtains only final values trough all its period of exploitation T, preserving its characteristic in the time t dual similar of the characteristic of the set point x^0, independently of purposive or accidental, external or internal influences is named **stability.**

According definition D2.2 a system to be stable there must to be valid the next:

Theorem T2.8. (necessary condition): An open system (see figure 1.5) to be stable it is necessary:

(2.18) $\lim(I^0 \pm \Delta I) \to I$,
by $I \neq \infty, t \to \infty$.

Proof: System on the figure 1.5 is invariant by saving the conditions according axiom A1.5. One from them is parameters R, L and C to be constant trough all working period of the system. It leads trivial to the conclusion that the invariant system is infinitely stable, because its output signal I obtains only finite values in infinite intervals of the time t. If the invariability be disturbed in finite values, i. e., impedance multitude $Z : \{R, L, C\}$ obtains the description:

(2.19) $Z : \{R \pm \Delta R, L \pm \Delta L, C \pm \Delta C\}$ and also is respected the condition:

(2.20) $\Delta R \neq \infty, \Delta L \neq \infty, \Delta C \neq \infty$,

there will be new invariants and the system also will be infinitely stable.

Theorem T2.9. (necessary condition): A closed system (see figure 2.5) to be stable it is necessary to be valid condition (2.16).

Proof: The closed system on figure 2.5 is invariant by same conditions as the open system on the figure 1.5. If its invariability is disturbed trough a linear change to the impedance multitude Z_{HO} of the object O according the rule:

(2.21) $Z_{HO} = Z_{HO} \pm \Delta Z_{HO}$ or

$Z_{HO} : \{k_{PO} \pm \Delta k_{PO}, T_{DO} \pm \Delta T_{DO}, \frac{1}{T_{IO} \pm \Delta T_{IO}}\}$,

and by that is valid the condition:

(2.22) $\Delta k_{PO} \neq \infty, \Delta T_{DO} \neq \infty$ and $\Delta T_{DI} \neq \infty$,

the system will be also infinitely stable.

Theorem T2.10. (Criterion of Lagrange): Stable is every system that consumes energy E_m whose variation δE_m:

(2.23) $\delta E_m \to 0, t \to \infty$

Remark: Criterion refers to distributed in the space energy that is also function of the time t. It is used in geology, seismology, designing of construction elements and activities etc., everywhere is valid space factor. Proof of the criterion is very complex and it is not necessary to apply, because discussed here systems are with concentrated parameters and the energy E_S is function only of the time t.

Definition D2.3. Variation of the concept energy E_S is the derivative dS/dt of the power $S(t)$ with that is supplied the object O.

Theorem T2.2. (sufficient condition): An open or closed system (see figure 1.5 or 2.5) it is sufficiently variation dS/dt of the energy E_S toward the object O to incline to zero trough all time t of the transitive process (mode), i. e.,

(2.24) $\frac{dS}{dt} \to 0, t \to \infty$

Proof: Energetic equation (2.1) describes the stable movement of the point with mass m. According first law of dynamics reactive force f of the point is equal of the active force of the energetic source f_G with effective value F_G, that presents the constant quality of action of the source. It is the reason that makes the point to move in stable state with the velocity $v(t)$, whose effective value V is constant. Therefore consumed by the point full energy

(2.25) $E_m = VFt = St$

is consumed by S=const. Then dS/dt=0.

Final result from the transformation (2.25) requires according the transitions (2.23) and (2.24)

the variation $\delta E_m(0)$ of the consumed by the point transitive energy $E_m(0)$ to be subordinate of the rule:

$$(2.26) \quad \delta E_m(0) = \lim \frac{dS(0)}{dt} \to 0, \text{ by } t \to \infty.$$

Then the point will pass from one stable state to other.

Theorem is proved.

D. Conclusions.

And so, it was proved that the energetic equations of every invariant system can be achieved with a universal method of approach by a dual similarity of the energetic equation (2.1) by invariant movement in resistive medium. The point is whit a constant mass m. If the mass is not constant as by movement of vehicle with proper energetic source (automobile, aeroplane, rocket etc.) system will be **invariant** by a constant **linear momentum** mv, but the invariability will be nonlinear. The method of approach to him will be dependent on properties of the non- invariability. A common method of approach is impossible.

Except to here describe classic (Newton's) invariability there exists:

1. Relativistic (Einstein's) invariability by that energy E necessary for acceleration of an elementary particle (material point) with mass by repose $m_0 = const.$ to velocity v near by the velocity of the light $c = const.$ is described by the equation:

$$(2.27) \quad E = mc^2,$$

where: $m = m_0 / \sqrt{1 - (v^2/c^2)}$.

2. Quantum invariability.

Energy E of a radiated quantum (photon) by an elementary particle is defined by equation:

$$(2.28) \quad E = (h/2\pi)\nu,$$

where: h is constant of Plank [Js], ν - frequency of the radiated photon [s^{-1}].

Equations (2.27) and (2.28) don't possess dual similarity between them as and with the equation (2.1) from classic invariability.

Axiom A2.5. Human controls a system by information perceived by his senses.

Axiom A2.6. Human senses can perceive information with velocities very lower toward the velocity c of the light.

Theorem T2.12. System for that the energetic information is not described dual through evident or not evident equations of classic (Newton's) mechanics is **not controlled by the human.**

Proof: In Newton's mechanics the functional dependence is described in absolute time t that requires every movement to be with velocities very lower toward the velocity c of the light. And according axioms A2.5 and A2.6 it is the unique condition by that is possible movement of the information about the process to be perceived by human to exercise a controlling action on him.

Theorem is proved.

And with that was proved and described the conditions by that the knowledge about Theory of Electricity can be successful leader of every specialist (engineer, physicist, chemist etc.) in the world of the invariant systems and their control as in control of material process from every nature.

REFERENCES

1. Brandiski K, J. Georgiev, V. Mladenov, Theoretical Electrical Engineering (Bulgarian text), Part One, IK KING, 2004
2. Brandiski K, J. Georgiev, V. Mladenov, Theoretical Electrical Engineering (Bulgarian text), Part Two, IK KING, 2005, 2008.
3. Manolov S. and other, High Mathematics (Bulgarian text), Part Four, Technica, 1974.
4. Petrov L., D. Baeva, Collection of Problems in High Mathematics (Bulgarian text), Module 6, Technical University – Sofia, FPMI, 2015.
5. Hadjidobrev P., Physics (Lectures - Bulgarian text), Technical University, College – Sliven, publication on IP address: http://tu-sliven.com/Studenti/UcebniM/Kol-Fizika-Lk.pdf
6. PID Controller – Wikipedia, the free encyclopedia.
7. Stankov M., Theory of Invariant Automatic Control, publication on IP address: http://framework.com/stankov/

Mag. Eng. Milan M. Stankov, *- Graduated from TU-Sofia in1967, major: Electrical Installations and Industrial Automatics. Working activity: projects of automation systems in chemical industry, energetic and transport. Research interests: Automatic control of technologic process with programmable controllers.*

e-mail: milan.stankov@mbox.contact.bg