Analysis of transient plane wave coupling to horizontal conductor in homogeneous lossy soil

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Modeling of transient behavior of wire conductors in presence of lossy soil has been a subject of great amount of research. This problem has been dealt with in different ways, from application of rigorous full-wave approaches based on Sommerfeld formulation to simplified models more suitable for practical engineering studies. This paper presents comparison of two distinct approximate models for analysis of transient plane wave coupling to horizontal wire conductor buried in homogeneous lossy soil. The first approach uses quasi-static approximation of corresponding Green's functions that arise in rigorous Sommerfeld integral based on image and complex image theory. The second approach uses transmission line theory where two formulations are compared. The first one is based on Sunde's integral whereas the second is based on simplified logarithmic expression for per unit length impedance. The authors compare the range of applicability of the two forms of image models and the two forms of transmission line models in practical EMC studies. The results are verified by comparison with Sommerfeld model on the basis of rms error of the current distribution with respect to frequency range from 10 kHz to 10 MHz.

Introduction

The electromagnetic field coupling to buried wires has been analyzed in many electromagnetic compatibility (EMC) studies [1]–[2] due to great practical interest. The analysis is often done by using approximate transmission line (TL) modeling due to the simplicity in implementation and use in existing software for high frequency analysis. However, this approach does not represent complete solution for the given problem since it doesn't include the radiation effects. On the other hand, the antenna theory approach based on rigorous electromagnetic theory [3] with at least approximations is often computationally inefficient. For that reason approximate methods within antenna theory models have been studied intensively [4]–[5], which are based on quasi-static image approximation. The results given in [6] show that significant differences between different models arise especially when analyzing buried wire conductors.

In this paper we compare the accuracy of two approximate approaches. The first approach is based on quasi-static (QS) image theory, and the second one is based on transmission line (TL) theory. Next, a comparison with respect to exact full-wave model will be done on the basis of by rms error of the current distribution with respect to frequency range from 10 kHz to 10 MHz. The main objective is to analyze the applicability domain of the analyzed models in practical EMC studies. The results are also compared by Numerical Electromagnetic Code (NEC) reflection coefficient solution.

Mathematical model

Consider a single *x*–directed horizontal conductor of radius *a*, and length *L* buried at depth *d* in finite conductive homogeneous soil, shown in Fig. 1. Here we assume uniform plane wave of normal incidence $\boldsymbol{E}^i = \hat{i}_x E_0 \exp(jk_0 z)$. The homogeneous lossy soil is characterized by permittivity $\varepsilon = \varepsilon_0\varepsilon_r$, permeability μ_0 and conductivity σ.

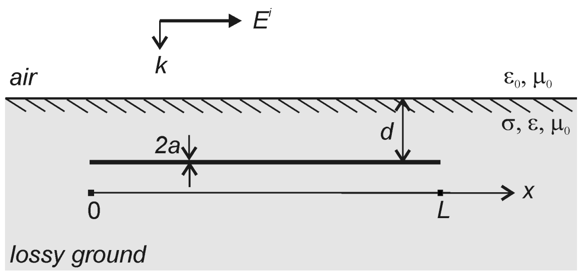

Fig.1. Horizontal wire conductor in lossy soil illuminated by a uniform plane wave of normal incidence.

To solve induced currents for a given problem we use moment method where the wire conductor is segmented in fictitious segments and the current distribution is approximated by overlapped triangular expansion functions. The current distribution is obtained by solving matrix equation $[I]=[Z]^{-1}\cdot[U]$, where [Z] is generalized impedance matrix, and [U] is excitation matrix.

The elements of [U] are determined by

$$U_n=\int_{l_n} E^{t01}dl_n \qquad E^{t01}=E_0 T^{01}\exp(-jk_1 d) \tag{1}$$

where E_{t01} is the electric field transmitted in lossy soil tangential to the wire conductor.

The elements of matrix [Z] are defined as mutual impedances between observation wire segment *m* due to filaments of current I_n and charge q_n induced along the axis of the source segment *n*

$$z_{mn}=\frac{1}{I_n}\int_{l_m}[\,j\omega\int_{l_n} G_A^{xx} I_n dl_{nx}-\nabla\int_{l_n} G_V q_n l_{nx}]dl_m \tag{2}$$

where G_A^{xx} is *x*–component of the dyadic Green's function for the magnetic vector potential, and G_V is scalar potential Green's function for a horizontal electric dipole HED in homogeneous lossy half-space.

Sommerfeld formulation

The Sommerfeld formulation represents most rigorous solution for the Green's functions for the given problem [7] where ($z=z'=d$)

$$G_A^{xx}=\frac{\mu_0}{2}[G_{dir}+I_1]=\frac{\mu_0}{2}[G_{dir}-KG_{img}+I_2] \qquad G_V=\frac{1}{2\underline{\varepsilon}}[G_{dir}+I_3]=\frac{1}{2\underline{\varepsilon}}[[G_{dir}+KG_{img}+I_4] \tag{3}$$

$$\begin{aligned}
I_1&=S_0\left\{R_{TE}\frac{\exp(-u_1|z+z'|)}{u_1}\right\}\\
I_2&=S_0\left\{\frac{2\exp(-u_1|z+z'|)}{u_1+u_0}\right\}\\
I_3&=S_0\left\{\frac{-u_1^2R_{TM}+k_1^2R_{TE}}{\lambda^2}\right\}\\
I_4&=S_0\left\{\frac{2k_1^2}{k_1^2u_0+k_0^2u_1}-\frac{2k_1^2}{u_1(k_1^2+k_0^2)}]\frac{\exp(-u_1|z+z'|)}{u_1}\right\}\\
R_{TE}&=\frac{u_1-u_0}{u_1+u_0} \quad R_{TM}=\frac{k_0^2u_1-k_1^2u_0}{k_0^2u_1+k_1^2u_0}\\
u_i&=\sqrt{\lambda^2-k_i^2} \quad \mathit{for}\ i=0,1\\
k_0^2&=\omega^2\mu_0\varepsilon_0 \quad k_1^2=k_0^2\underline{\varepsilon}_r\\
\underline{\varepsilon}_r&=\varepsilon_r-\frac{j\sigma}{\omega\varepsilon_0} \quad \underline{\varepsilon}=\varepsilon_0\underline{\varepsilon}_r
\end{aligned} \tag{4}$$

here $G_{dir}=\frac{\exp(-jk_1R_{dir})}{R_{dir}}$ and $G_{img}=\frac{\exp(-jk_1R_{img})}{R_{img}}$ are direct and image terms that represent respectively a spherical wave that arises in the case when the HED source and its image are in infinite homogeneous medium with characteristics of the soil with propagation constant k_1, and R_{dir} and R_{img} are distances between the source HED and its image to the observation point. Terms $S_0\{\}=\frac{1}{2\pi}\int_0^\infty\{\cdot\}J_0(\lambda\rho)\lambda d\lambda$ represent Sommerfeld integrals which are solved by direct numerical integration.

Quasi-static formulations

The quasi-static (QS) image models are based on approximation of the kernel in Sommerfeld integrals that arise in exact formulation of the problem [4].

The first model (denoted as "QS-img") is based on approximation of kernels in I_1 and I_3 by using the following approximation of the reflection coefficients, $R_{TE}\to 0$ and $R_{TM}\to\frac{k_0^2-k_1^2}{k_0^2+k_1^2}=-K$, where $u_1\sim u_0$ due to $\lambda^2>>k_i^2$ for $i=0,1$ as $\omega\to 0$ and $k_0^2\to 0$.

$R_{TM}\to -K$ is a key simplification because *K* is a constant in the spectral domain and can be extracted from the integrals, which enables the derivation of closed-form solutions of the integrals.

This leads to approximate formulations of the Green's functions

$$G_A^{xx}\approx\frac{\mu_0}{2}G_{dir} \qquad G_V\approx\frac{1}{2\underline{\varepsilon}}[G_{dir}+KG_{img}]. \tag{5}$$

The second approach (denoted as "QS-cmplx.img") uses Wait-Spies [9] and Bannister's extended image approximation [10] in order to simplify kernels in I_2 and I_4 [5].

This approximation is based on assumption $u_0\sim\lambda$ and $u_1\neq u_0$ as $\omega\to 0$ that leads to the following approximation $\frac{2\lambda}{u_1+\lambda}\approx 1-\exp(-\lambda d_C)$ where $d_C=2/jk_1$ is complex depth [10].

In order to obtain closed form solution of approximate integrals we use approximation [10] $\exp(-u_1|z+z'|)=\exp(-jk_1A|z+z'|)\exp(-\lambda B|z+z'|)$ where *A* and *B* are constants [10].

Thus, we obtain a second set of approximate Green's functions as follows

$$G_A^{xx} \approx \frac{\mu_0}{2}\left[G_{dir} + (K-2)G_{img} + \frac{\exp(-jk_1A|z+z'|)}{2\pi R_1} - \frac{\exp(-jk_1A|z+z'|)}{2\pi R_2}\right] \tag{6}$$

$$G_V \approx \frac{1}{2\underline{\varepsilon}}\left[G_{dir} + (K-2)G_{img} + \frac{2\exp(-jk_1A|z+z'|)}{2\pi R_1}\right]$$

where

$$R_1 = \sqrt{\rho^2 + \left(B|z+z'|^2\right)} \qquad R_2 = \sqrt{\rho^2 + \left(d_C + B|z+z'|^2\right)}.$$

Transmission line formulations

The transmission line (TL) equations for a buried horizontal wire conductor are expressed in terms of voltage $V(x)$ and current $I(x)$ induced along the conductor

$$\begin{aligned} &\frac{\partial V(x)}{\partial x} + ZI(x) = E^{t01}(x,-d) \\ &\frac{\partial I(x)}{\partial x} + YV(x) = 0 \end{aligned}, \tag{7}$$

where Z and Y are respectively per unit length soil impedance and soil admittance.

The current distribution is obtained by

$$\begin{aligned} I(x) = &\frac{E^{t01}}{Z_0}\frac{(1-\exp(\gamma_1 L))\exp(-\gamma_1 x)}{(\exp(\gamma_1 L)-\exp(-\gamma_1 L))} \\ &+ \frac{-E^{t01}}{Z_0}\frac{(1-\exp(-\gamma_1 L))\exp(\gamma_1 x)}{(\exp(\gamma_1 L)-\exp(-\gamma_1 L))} + \frac{E^{t01}}{Z_0} \end{aligned} \tag{8}$$

where $Z_0 = \sqrt{Z/Y}$ is the characteristic impedance.

Here we compare two TL models to calculate the induced currents.

The first model is based on Pollaczek formulation for per unit length impedance Z and admittance Y in integral form (denoted by “TL-int”)

$$\begin{aligned} &Z = \frac{j\omega\mu_0}{2\pi}\left[K_0(\gamma_1 a) - K_0\left(\gamma_1\sqrt{a^2+4d^2}\right) + I_S\right] \\ &I_S = 2\int_0^\infty \frac{\exp\left(-2d\sqrt{\lambda^2+\gamma_1^2}\right)\cos(\lambda a)}{\lambda + \sqrt{\lambda^2+\gamma_1^2}} d\lambda \end{aligned} \tag{9}$$

whereas the second model uses logarithmic simplified formulation [8] (denoted by “TL-log”)

$$Z \approx \frac{j\omega\mu_0}{2\pi}\ln\left(\frac{1+\gamma_1 a}{\gamma_1 a}\right). \tag{10}$$

In both cases the soil admittance is calculated by

$$Y = \frac{\gamma_1^2}{Z}. \tag{11}$$

Numerical results

In this paper we compare the currents induced along the buried conductors due to illumination by transmitted electric field. The currents are calculated by using the approach based on Sommerfeld formulation, the two QS approximate formulations, and the two TL formulations. The results are compared also with those obtained by using the Numerical Electromagnetic Code (NEC) reflection coefficient approximation.

The studied cases are: L = 20-m (short conductor) and L = 100-m (long conductor), with radius a = 0.007 m at depth d = 1 m and d = 0.5 m. We use two values for the soil conductivity σ = 0.01 S/m (medium) and σ = 0.1 S/m (high). The relative permittivity of the ground is fixed at ε_r = 10. The electric field in the air is assumed E_0 = 1 V/m in frequency range from 10 kHz to 10 MHz.

The accuracy of the proposed approximate models is analyzed in frequency domain by comparing the induced currents along the conductor with respect to the results obtained by using rigorous Sommerfeld formulation. We calculate the following normalized RMS error

$$\varepsilon_{rms} = \left[\frac{\sum_{i=1}^{N}\left|\underline{I}_{Ei} - \underline{I}_{approxi}\right|^2}{\sum_{i=1}^{N}\left|\underline{I}_{approxi}\right|^2}\right]^{\frac{1}{2}} \tag{12}$$

where $\underline{I}_{Ei}$ and $\underline{I}_{approxi}$ are phasors of the current samples along the wire computed by the Sommerfeld formulation and the approximate QS-img, QS cmplx.img., TL-int and TL-log models respectively, and N is number of samples.

Short 20-m conductor

Fig. 2 shows the current magnitude induced along a 20-m horizontal conductor buried at 1 m in homogeneous lossy soil (ε_r = 10; σ = 0.01 S/m) at 1 MHz and 10 MHz. The corresponding ε_{rms} error is shown in Fig. 3. In Fig. 4 it may be observed the ε_{rms} error calculated for high soil conductivity (σ = 0.1 S/m), whereas in Fig. 5 it may be observed the influence of the conductor depth on the ε_{rms} error (here d = 0.5 m).

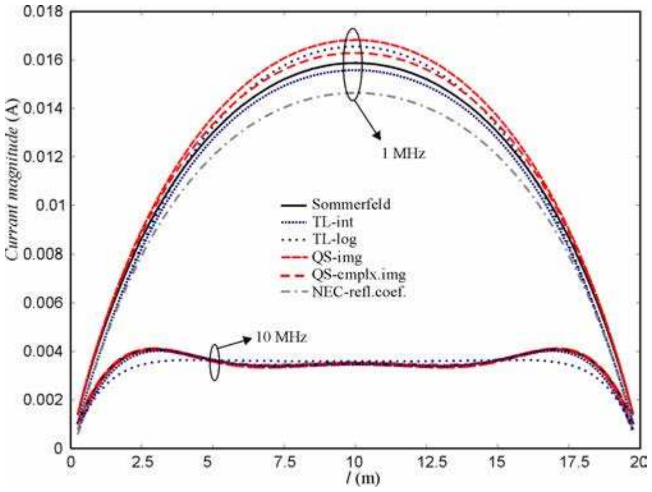

Fig.2. Current induced along 20-m conductor in lossy soil due to electric field.

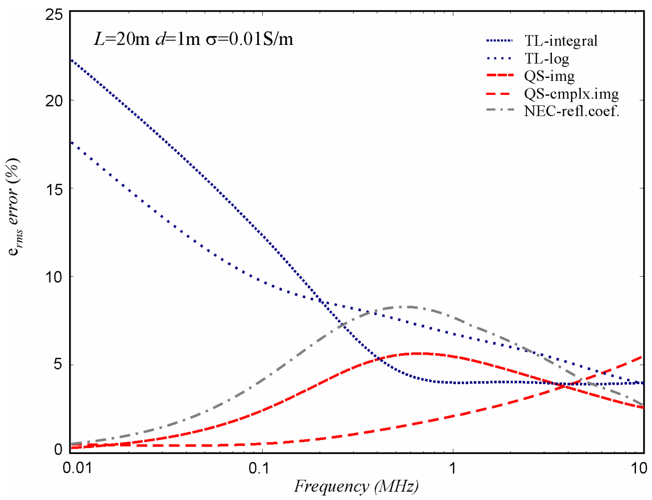

Fig.3. ε_{rms} error of the current along 20-m conductor (d = 1 m; ε_r = 10; σ = 0.01 S/m).

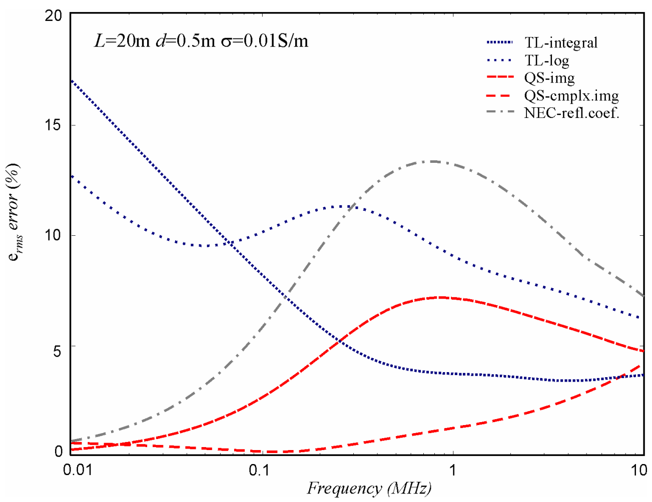

Fig.4. ε_{rms} error of the current along 20-m conductor (d = 1 m; ε_r = 10; σ = 0.1 S/m).

As may be observed, both TL-int and TL-models introduce significant ε_{rms} error (> 10%) in the lower frequency range whereas for frequencies above 1 MHz this error is about 5%. On the other hand, QS-img and QS-cmplx.img models show very good agreement with the results obtained by Sommerfeld formulation. The ε_{rms} error introduced when using NEC reflection coefficient method is larger. Also, the results show that better agreement is obtained when the soil conductivity is high. Fig. 5 shows that when the conductor depth decreases the accuracy of TL models increases, whereas the QS image models show slightly worse results.

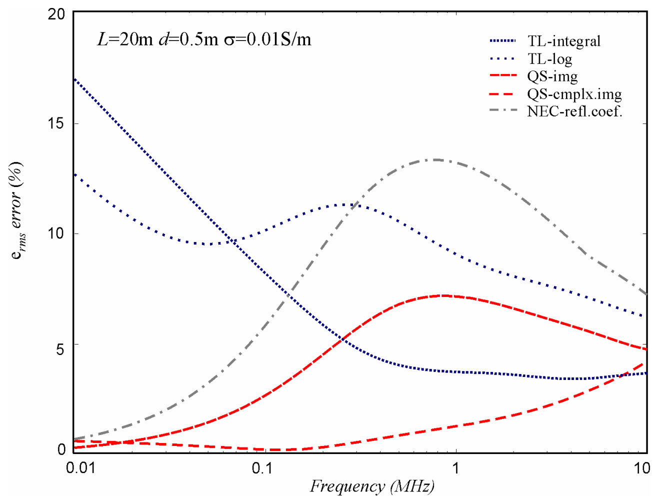

Fig.5. ε_{rms} error of the current along 20-m conductor (d = 0.5 m; ε_r = 10; σ = 0.01 S/m).

Long 100-m conductor

Similarly as previously, Fig. 6 represents the induced current along a 100-m horizontal conductor at 1 m in homogeneous lossy soil (ε_r = 10; σ = 0.01 S/m) at 1 MHz and 10 MHz. The corresponding ε_{rms} error is shown in Fig. 7. As may be observed, the two TL models show better agreement when applied for long conductors. However, higher ε_{rms} error (> 5 %) is still observed at low frequencies. The results obtained by QS-img and QS-cmplx.img models are also in good agreement with respect to Sommerfeld formulation model, it may be observed that the max ε_{rms} error is about 5% in all frequency range. In Fig. 8 it may be observed the ε_{rms} error in case when the soil conductivity is high (σ = 0.1 S/m). As may be observed, when the soil conductivity increases the accuracy of all approximate models is improved, i.e. ε_{rms} error decreases. Finally, in Fig. 9 it may be observed the influence of the smaller conductor depth (d = 0.5 m) on the ε_{rms} error.

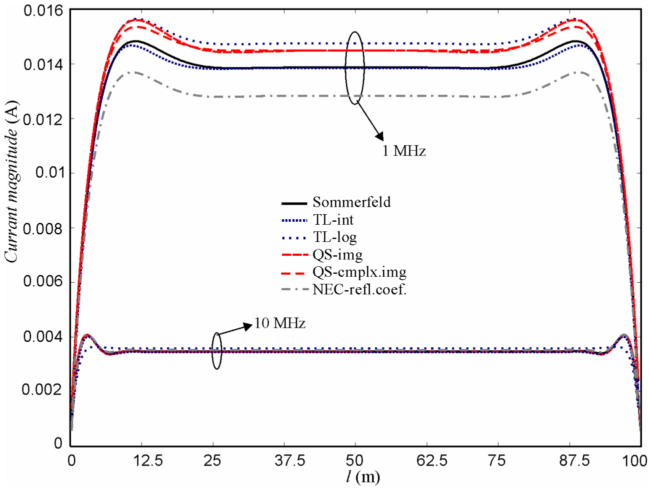

Fig.6. Current induced along 100-m conductor in lossy soil due to electric field.

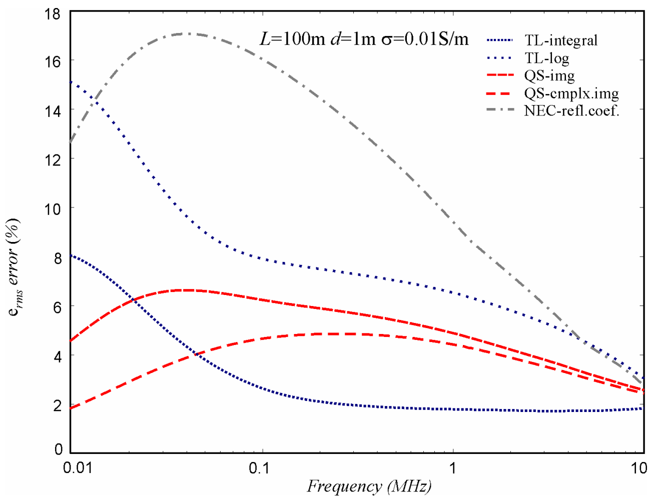

Fig.7. ε_{rms} error of the current along 100-m conductor (d = 1 m; ε_r = 10; σ = 0.01 S/m).

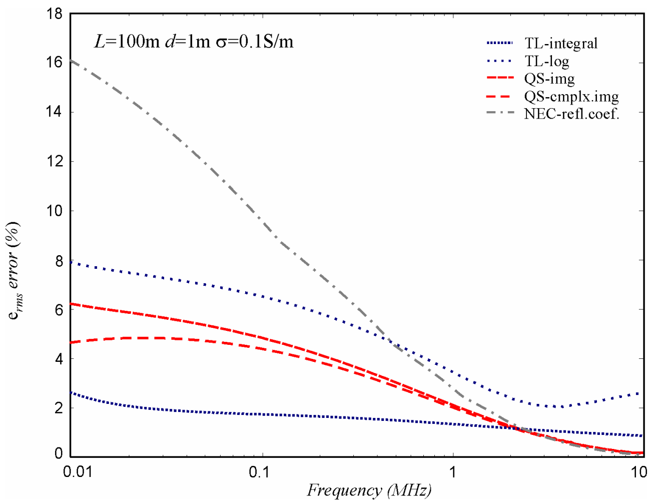

Fig.8. ε_{rms} error of the current along 100-m conductor (d = 1 m; ε_r = 10; σ = 0.1 S/m).

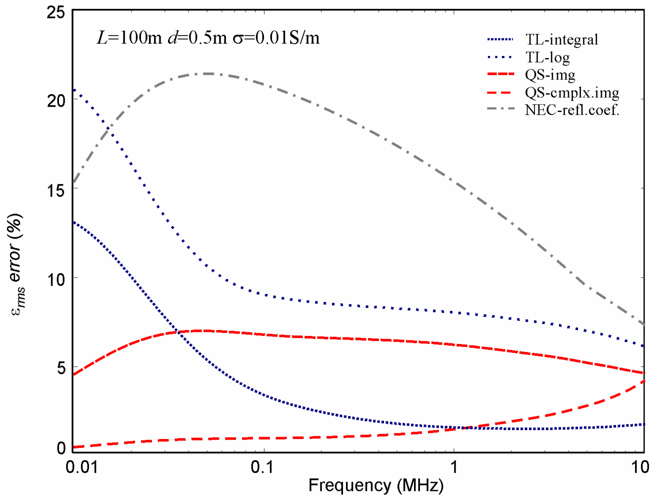

Fig.9. ε_{rms} error of the current along 100-m conductor (d = 0.5 m; ε_r = 10; σ = 0.01 S/m).

Conclusion

In the paper, the authors analyze two distinct approximate formulations (image theory and transmission line theory) in modeling transient coupling to horizontal wire conductor buried in homogeneous lossy soil. The simulation results of the induced currents along a 20-m and a 100-m wire conductor may be summarized in:

The QS image/complex image models are very accurate and lead to generally small ε_{rms} error practically in all studied frequency range. However, the ε_{rms} error increases when the soil conductivity decreases. The results show that QS complex image approximation represents best approximation with about 5% ε_{rms} error in all studied cases. Only exception is in case of small value of soil conductivity and frequencies above 1 MHz when it may be observed ε_{rms} error errors higher than 5%.

The TL integral/log models show good agreement of the currents in case when the conductors are long and the ground conductivity is high. However, the analyzed TL integral/log models introduce significant ε_{rms} error (> 10%) in the lower frequency range.

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