Analytical approach for calculation of the electric field induced by an axisymmetric current exciter

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In the paper is presented an analytical approach for calculation of the electric field induced by an axisymmetric current exciter. The region in which the field is determined may have an arbitrary shape. The approach is based on the well-known theoretical results referring to the magnetic vector potential of a circuit current contour. Provided that the environment is linear, the "superposition" principle is used. A practical example of a flat heating induction system with an inductor shaped as a flat spiral coil is regarded. The inductor is situated at a distance from an aluminum disc detail. An analytical model of the inducted electrical field in the disc is formulated and after that a model of the current distribution in the detail is given. The results are compared with those obtained by using a model of the same system represented by inductively connected circuits, as well as with the experimental results.

Аналитично изследване на електрическо поле, индуктирано от осесиметричен токов възбудител (Илонка Лилянова)*. В статията е предложен аналитичен подход за изчисляване на електрическо поле, индуктирано от осесиметричен токов възбудител. Областта, в която се определя полето може да има произволна форма. Подходът се базира на известни теоретични изводи за магнитния вектор-потенциал на кръгов токов контур. При условие, че средата е линейна, се използва принципът с наслагването. Представен е приложен пример на плоска индукционна система, с индуктор във формата на плоска спирала. Индукторът е разположен на разстояние от дисков алуминиев детайл. Формулиран е аналитичен модел на индуктираното в диска електрическо поле и оттам на разпределението на токовете в детайла. Резултатите са сравнени с тези, получени с използването на модел на системата, представен от индуктивно свързани вериги, както и с резултати, получени експериментално.*

Introduction

During the induction heating (IH) the distantly excited electromagnetic energy penetrates directly into the heated detail. The process is characterized with: very high concentration of energy in the heated material, non-contact of the impact, reliability at action, easy regulation, does not cause any pollution. Apart from its traditional use in the heat-treating of metals [1], [2], it is also used in fluids heating [3], in demagnetizing of ferromagnetic details; in processing compact ceramic materials [2], in cooking[5] and in medicine[6]. The research of the literature sources shows that the problems of induction heating continue to be one of the main interested topics not only needing more accurate results, but also developing inductors for special types of temperature treatment. Some of them have very specific shapes [7], [8]. The wide variety of practical uses of IH demands finding solutions to the very important inverse problems related to boosting the efficiency of the technic ware and the processes in them. This requires the existence of the precise computing models, especially with respect to the electromagnetic field as the main origin of the heating [9].

The present paper suggests an approach for analytical research of the electromagnetic field "excited" by an axis-symmetric current exciter. It is also possible to find a solution for the more general case of the system in which the "exciting" circular contours with known currents in them are orientated differently in the space. The approach is based on well-known theoretical conslusions for the magentic vector potential determined by a circular current contour [10], [11]. Summarising geometrically the created vector potentials at a particular point, the approach can be used for analysing the electromagnetic field in a space region with an arbitrary geometry. The reliability of the research is proved by a practical example in a linear environment. Thus at a flat heating induction system the radial distribution of the current density and the induced currents in aluminium disc detail are computed. The results are compared with those obtained by studying another model of the system presented by a inductively connected electrical circuits [2], as well as by experimental results [3].

Electrical field excited by a circular current contour

The proposed model is named as model 1. To compute the electromagnetic field of the currents flowing in the contours with complicated shapes is a rather difficult problem. In this case all quantities determining the field are functions of the spatial coordinates. The general method of a solution in a homogeneous environment consists in finding the magnetic vector potential supposing that the distribution of the exciting currents is known.

By using this method the derivative fields – magnetic inductions and induced electrical field (in the cases when the current is dependent on time) are computed later. Taking into account that every axis-symmetric current exciter could be considered as a group of circular contours with the same currents, it is clear that in order to determine the magnetic vector potential and the additional fields the problem must be solved in the space [10]. Since the magnetic vector potential $\vec{A}$ is collinear with the current vector density $\vec{J}$ of the exciter, the vector lines of $\vec{A} = A_\alpha \vec{e}_\alpha$ lie in planes, parallel to the exciting contours and they look like concentric circles with a centers at the axis of these contours (Fig. 1).

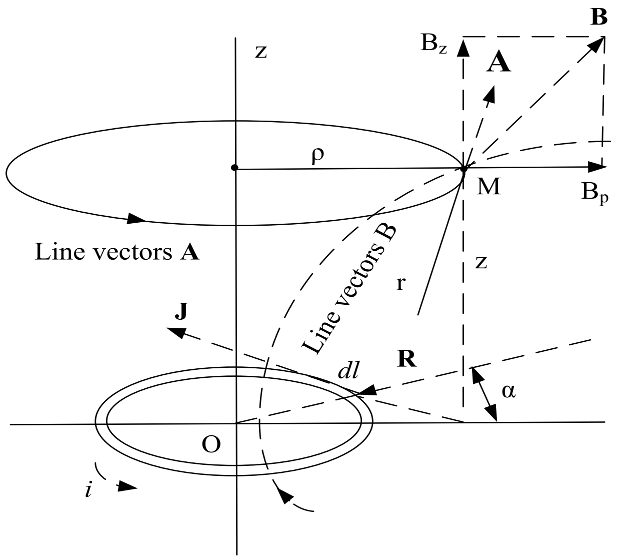

Fig.1. Field of circular contour.

The physical field structure of a single current contour with current i requires the study in a cylindrical coordinate system ρ, α, z.

In the cylindrical coordinate system of Fig. 1 the magnetic vector-potential is determined at point M. The vector line $\vec{A}$, through the point M lies in the plane parallel to the plane in which the exciting current contour is situated. The origin of the coordinate system is point O. It is the common center of all circular windings (current contours), equivalently replacing the inductor. The axis $\vec{e}_z$ is perpendicular to the plane of the current contours. It is considered that the sizes of the cross section *s* of each current contour are much smaller than its radius R. Furthermore, the point M is placed at a much bigger distance from the exciting contour than each linear size of its cross section S. Those circumstances make it possible the magnetic vector-potential $\vec{A}$ at the arbitrary point M to be defined as excited by a linear current contour G:

(1) $$\vec{A} = \frac{\mu . i}{4\pi} \oint_G \frac{d\vec{l}}{r}$$

The vector-potential has only a tangential component $A = A_\alpha$ which at point M is defined by the following expression:

(2) $$A = \frac{\mu . i}{2\pi} \sqrt{\frac{R}{\rho}} \left[\left(\frac{2}{k} - k \right) K - \frac{2}{k} E \right]$$

The computation is implemented by finding the exact values of the two elliptical integrals K and E, from I and II order respectively, or by using rows to find an approximate value. The modulus *k* depends on the geometric sizes and it is defined by the following expression:

(3) $$k = \sqrt{\frac{4R\rho}{z^2 + (R+\rho)^2}}$$

Due to the field dependencies between the vectors of the electromagnetic field, between the electrical intensity $\vec{E}$, the magnetic induction $\vec{B}$ and the magnetic vector potential $\vec{A}$, the following dependencies are used:

(4) $$rot\,\vec{E} = -\frac{d\vec{B}}{dt} \quad \text{и} \quad \vec{B} = rot\,\vec{A},$$

where:

(5) $$rot\,(\vec{E} + \frac{d\vec{A}}{dt}) = 0$$

and as a result:

(6) $$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -grad\,V.$$

Here, the quantity V is a scalar electric potential. Due to the fact that the electrical field in the detail is defined only by the eddy current, an induction field $\vec{E}$ can be found by the expression:

(7) $$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

At a sine mode (7) is presented in a complex form:

(8) $$\dot{E} = -j\omega\dot{A} \quad \dot{E} = -j\omega\dot{A}.$$

Since the components of an electrical field $\vec{E}$ are defined by those of the vector potential $\vec{A}$, it follows:

(9) $$\vec{E} = E\,\vec{e}_{\alpha}$$

Applying the Ohm's law in a differential form $\vec{J} = \gamma\,\vec{E}$, it is concluded that the density $\vec{J}$ of the induced currents can be computed in every detail point, as well as in the detail as a whole. In order to do that it is used the superposition principle (the environment is linear). Since the detail is circular and the field $\vec{E}$ is symmetric, it is enough to define a radial distribution $E_{\alpha}(r)$ and respectively $J_{\alpha}(r) = \gamma\,E_{\alpha}(r)$.

The induced electric field in the studied object (model 1)

The investigated real system is shown in Fig. 2.

Fig.2 View of the induction system with 7 sections.

The exciter is a single-layer inductor with seven sections each consisting of 13 circular windings. It has an inner diameter 0,006m and an outer diameter – 0,48m. It is made of a PSD 1,2x10mm conductor. Parallel to the inductor a non-ferromagnetic aluminium detail is situated, where the eddy current electrical field is induced. The system works under laboratory conditions. It is supplied by a network with industrial frequency and lower voltage.

The inductor is presented as a group of K current contours. Each contour has a size in radial direction less than 2 mm (the cross section width). The detail which is also circular is presented also as a group of current contours, but contours of the induced currents. Thus in a detail current contour with arbitrary number n the induced current density is defined by the algebraic sum $J_n = \sum_{k=1}^{K} \gamma\,\omega\,A_{kn}$. Here γ is the specific electric conductivity of the material, ω - circular frequency, and A_{kn} is the magnetic vector potential, induced at n-th current contour of the detial from a k-th exciter current contour of the inductor.

The current I at each point of the detail is calculated by the expression $I = J.\Delta s$, where Δs is the cross section of the respective elementary current contour of the detail. It is defined by the thickness (the height) of the detail and the radial cross sectional size of the corresponding current contour.

The height of the inductor circular winding is not included directly in the calculations, however it must be noted that the smaller it is, the more precise the calculations are. Thus, the geometry of the exciting current contour is close to the ideal one. For these reasons the inductor is presented as composed of five layers along its height. In this manner at a particular point of the detail the vector potential components are summed from all the layers of the inductor. The different distance between the layers and the detail is taken into consideration. The computations are done using the programming environment of MATLAB.

Model 2 of the induction system

Model 2 is built on the basis of the electrical circuit theory. It is a group of inductively connected electrical circuits, equivalently replacing the elements of the induction system.

The electromagnetic device is replaced schematically with inductively connected contours (circuits). In the case of a flat inductor-detail the system replacing electrical scheme looks like the one shown in Fig. 3 [12].

The parameters of the inductor are presented by the active resistance R_{ind} and the self-inductance L_{ind}, and by R_{det} and L_{det} the respective parameters of the heated detail (working body) are defined. The number of sections, the way they are turned on, the active and the inductive resistances are specified for the corresponding inductor model [1].

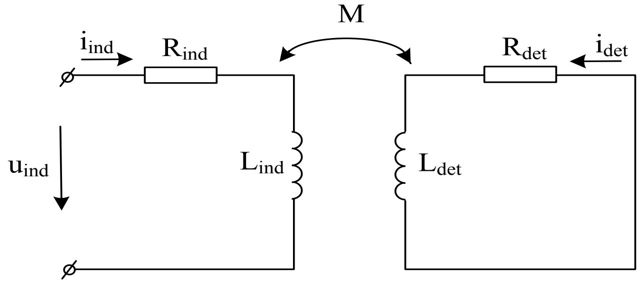

Fig.3.Electrical scheme.

The main benefit of the analysis made by using model 2 is the possibility the problem to be solved for a particular value of the supplying voltage. The electromagnetic field looking at is regarded as a quasi-steady-state. It is considered that all currents and voltages are changing accordingly to a harmonic law. The magnetic environment is linear and the dependency magnetic flux density–magnetic field intensity $B = \mu.H$ is valid. The computations are done with the help of complexes.

By using combinations of theoretical methods the parameters of the electrical circuits equivalently replacing the induction system are computed, the current density distribution is found and respectively the currents in the detail are found. These are the quantities to be defined by all physical characteristics of the electromagnetic and heating process.

Experimental studies

A great number of experiments are implemented by measuring the electromotive voltages inducted in the measuring drill coils [13]. They are singular circular coils, concentrically situated as each first and last winding of the sections in the inductor. The obtained experimental results have been additionally analysed. Thus, on the basis of the obtained experimental material, the following quantities have been determined: the distribution of the currents in the respective working body (detail) and the distribution of the inductions in the space between the detail and the inductor.

The electromotive voltage inducted in the i^{th} drill coil E_{ci}, provides information about the common magnetic stream Φ_i through the area covered by this drill coil.

(10) $$\Phi_i = \frac{E_{ci}}{4{,}44.f.w_i} = \frac{E_{ci}}{222}$$

$$w_i = 1, \quad f = 50Hz.$$

On the basis of the measured electromotive force (EMF) in the drill coils, using the respective linear transformations (due to the linearity of the environment in which the magnetic field is spread and the minimal distance between the inductor and the detail), finally experimentally is defined the distribution of the currents in the detail [12]. Since the relations are based only on the proportions, the distribution of the currents, this one found by computations based on the theoretical model and the one obtained by the use of the experimental results for the induced EMF, can be compared only if they are reduced to relative units. For the theoretical results, including those obtained by using model 1, the transformation in relative units for the system of currents in the detail removes the inconvenience of considering the dependency of the specific aluminium conductivity on the temperature. The latter prevents the search of accurate absolute values of the currents in the detail. In order to do this, experimentally, as well as theoretically, the obtained curve is normalised with respect to the maximum of current values in the respective series of the results.

Computational results and comparisons

Comparison between the results obtained from the two theoretical models 1 and 2. The results for the currents, obtained by using model 1 – the proposed new one and the model 2 – with electrical replacing schemes of the inductor and the detail are compared according to the absolute values of the currents in the detail. The comparison is presented graphically. The currents computed on the basis of the model 1, are marked in the graphics as I_{new}, and those computed on the basis of the model 2 are marked as I_d. The relative percentage deviation δ between the values obtained by the two methods is found as follows:

(11) $$\delta,\% = \frac{I_d - I_{new}}{I_d}.100.$$

Considering that all the sequentially connected sections of the inductor are turned on, the distribution of the currents in the aluminium disc is shown in Fig. 4. *T*he big differences between the two types of the results are in the circular zone within the limits between $0.03 \le r \le 0.06\,\mathrm{m}$, just rights outside the opening of the inductor. The explanation is that this zone is close to the inner opening of the inductor and it is influenced by the end effect.

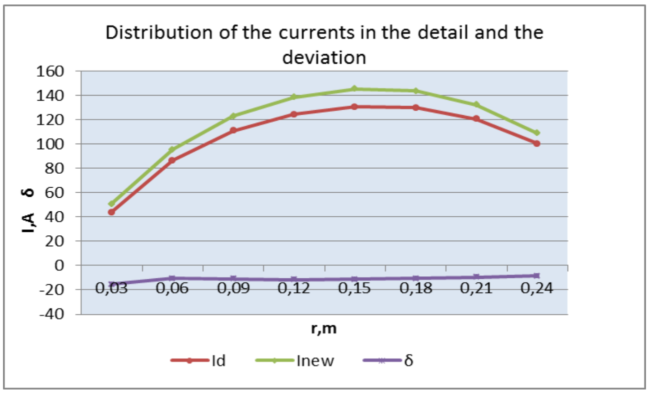

Fig.4. Distribution of the currents in the detail and the relative deviation between them.

It has to be noticed that radially in a quite wide range $0.07 \leq r \leq 0.21$ m, the absolute value of the relative deviations is less than 11,5%. The comparison confirms the fact that the proposed theoretical model is reliable with respect to accuracy and can be used for additional research, including optimisation of the electromagnetic system. The latter fact is supported by the result that in the region with a maximal value of the current, therefore with maximal temperatures, the deviation between the currents is smaller.

The distribution of the electric field in the aluminium disc when all the sections of the inductor, apart from the 5th one, are sequentially connected is shown in Fig.5.

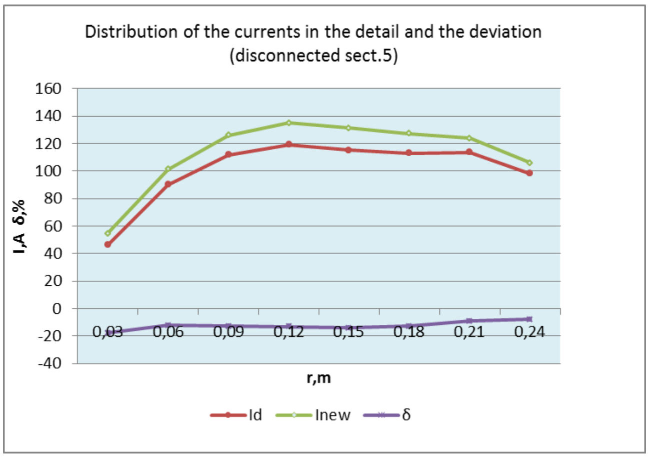

Fig.5 The distribution of the currents in the detail and the relative difference between them δ.

The biggest deviations between the currents using the two types of models are seen in the circular belt within the limits of $0.03 \leq r \leq 0.06$ m, just right outside the opening of the inductor, as well as at the distance 0.15m from the centre of the detail. This is the place of the missing 5th section. At this place the percentage deviation goes close to 14%.

A comparison of the results between the newly proposed theoretical model and those obtained experimentally.

A better guarantee for reliability should be looked for in the comparison with the experimentally obtained results [13]. The comparison is made in relative units. Theoretically, as well as experimentally, the obtained curve is normalised according to the maximum of the current values in the respective series of results.

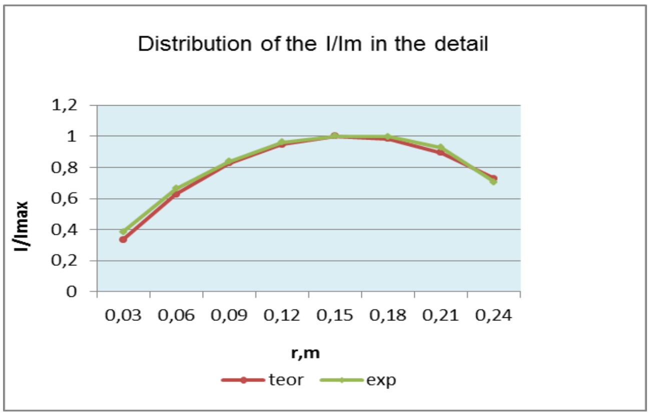

Fig.6. Distribution of the I/I_m in the detail.

Theoretically and experimentally obtained graphics in relative units I/I_m, when all sections of the inductor are turned on, are shown in Fig. 6. With I_m the maximal value of the current in the detail is expressed.

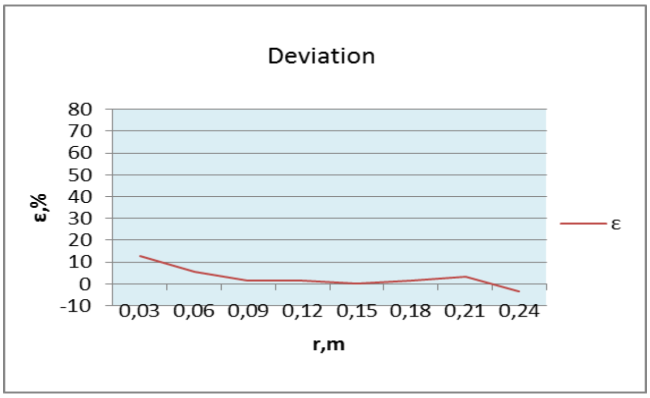

Fig.7. Percentage deviation between the results.

The relative value of the deviation ε, % between the theoretically (model 1) and the experimentally obtained graphics is shown in Fig.7.

$$\varepsilon,\% = \frac{(I/I_m)_{\exp} - (I/I_m)_{teor}}{(I/I_m)_{\exp}}.100 \qquad (12)$$

In the most temperature-loaded interval $0.05 \le r \le 0.24\,\text{m}$, with the maximum value I_m of the current, the theoretical and experimental results differ within the limits $\varepsilon,\% \le 5\%$.

Series-connected sections 1, 2, 3, 4, 6, 7 and disconnected section 5

Distribution of the ratio I/I_m in the detail (model 1) and the experimentally defined one in the detail with the 5th section of the inductor missing is shown in Fig.8.

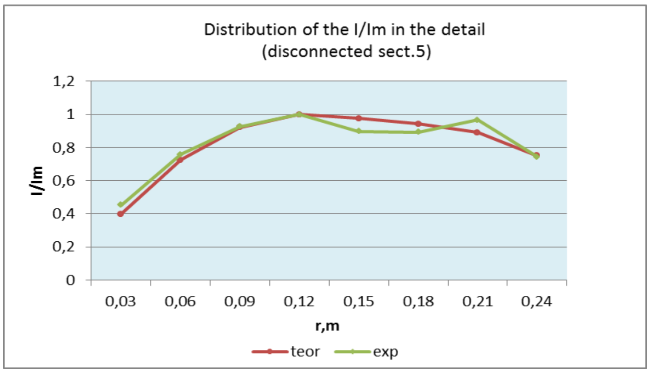

Fig.8. Distribution of the I/I_m in the detail.

In this case the maximal deviation between the results is in the region of the missing 5th section as well. Despite that, the respective percentage deviation is less than 7,5%.

Conclusion

A field approach for analytical research of devices with an axis-symmetric exciter (inductor) which can be presented by a system of circular current contours is suggested.

The newly proposed analytical approach provides guaranteed accuracy, verified by the presented practical example. This approach can be used for additional theoretical research of induction systems with the purpose of raising their efficiency. With a slightly more complicated procedure (applying vector sum in observed points) the method is applicable for defining the field in a spatial region with an arbitrary geometry and also in the case of an exciter, presenting a system of circular current contours which are situated at a planes diverged one to another at different angles.

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