A model of a single-phase synchronous generator with rare earth magnets

Nikola Georgiev

A model of a single-phase synchronous generator with rare earth magnets (Nikola Georgiev). *This paper makes a study of a single-phase synchronous 16-pole generator with axial magnetic field and rare earth magnets. A model of the synchronous generator has first been developed, taking into consideration its geometric dimensions, the materials it is made of, as well as the parameters of the windings and of its rare earth magnets. By means of the developed model, the voltages at idle running have been calculated for the case of connecting both of its sections in parallel. Replacement schemes of the generator for the modes of idle running and active load have been worked out. By means of them the output electrical parameters (voltage, current and power) of the 16-pole generator have been calculated. Experimental studies of the output electrical parameters of the real single-phase generator under the modes of idle running and active load have been conducted. The basic output characteristics, obtained from the model, have been compared to those, from the experiments.*

Модел на монофазен синхронен генератор с редкоземни магнити (Никола Георгиев). *Изследва се монофазен синхронен шестнадесет полюсен генератор с аксиално магнитно поле и редкоземни магнити. Първоначално е получен модел на синхронния генератор, в който са отчетени геометричните му размери, материалите, от които е изработен, както и параметрите на намотките и редкоземните му магнити. С помощта на полученият модел са изчислени напреженията на празен ход, при включени двете му секции в паралел. Съставени са заместващи схеми на генератора за режимите на празен ход и при активен товар. С тяхна помощ са изчислени изходните електрически параметри – напрежение, ток и мощност, на шестнадесетполюсния генератор. Направени са експериментални изследвания на изходните електрически параметри на разглеждания монофазен генератор в режими на празен ход и при активен товар. Сравнени са основните изходни характеристики, получени от модела с тези от извършените експериментални измервания.*

Introduction

The generators with permanent magnets and axial magnetic field are with a simple and reliable construction and high power per unit of weight. Their other advantages are the absence of excitation winding and current, which leads to high efficiency in operation. Additional improvement of their characteristics is achieved by using high power permanent magnets, such as the rare earth magnets NdFeB [1].

Initially, the generators with rare earth magnets and axial magnetic field were unilateral with one rotor and one stator. Then bilateral generators with one rotor and two stators or with two rotors and one stator appeared [2].

A model of a unilateral generator is considered in [3], while [4] discusses a bilateral generator with two rotors and one stator, where the total stator magnetic flux is calculated by means of the theorem of Stokes. In [5], again, a single-phase generator is examined, which is bilateral with two rotors and one stator, and the flux linkage is calculated by the finite element method, and then the r.m.s. value of the phase electromotive voltage is calculated. Similar are the models in [6], where the magnetic flux and the instantaneous value of the electromotive voltage are calculated, as well as in [7], where the r.m.s. value of the stator electromotive voltage is defined.

A low power single-phase synchronous 16-pole generator with axial magnetic field and rare earth magnets, composed of one rotor and two stators, has been modeled and studied in this paper.

Exposition

The paper presents the mathematical model of the considered synchronous 16-pole generator.

Fig.1 shows the construction scheme of the considered two-section generator and a view from the left, with the following notations used in the scheme: 1 - the aluminum rotor; 2 - the rare earth magnets; 3 - the windings for sections A and B of the generator; 4 – the steel stators; 5 – the shaft of the rotor.

Fig. 2 shows a top view of the construction scheme of the generator and the magnetic circuit.

The magnet reluctances of both the air gap $R_{\mu B}$ and the steel stator $R_{\mu CT}$ are presented in the two figures by a dotted line.

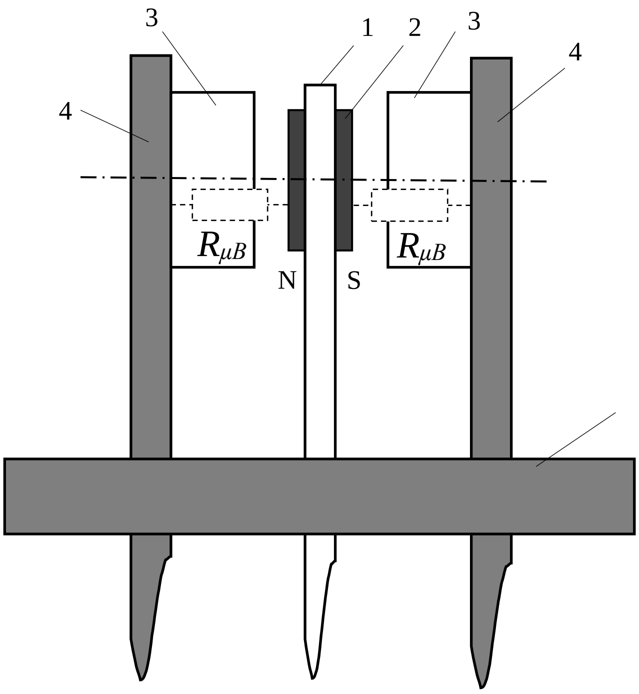

Fig. 1. Constructive scheme of the two-section generator and a view from the left

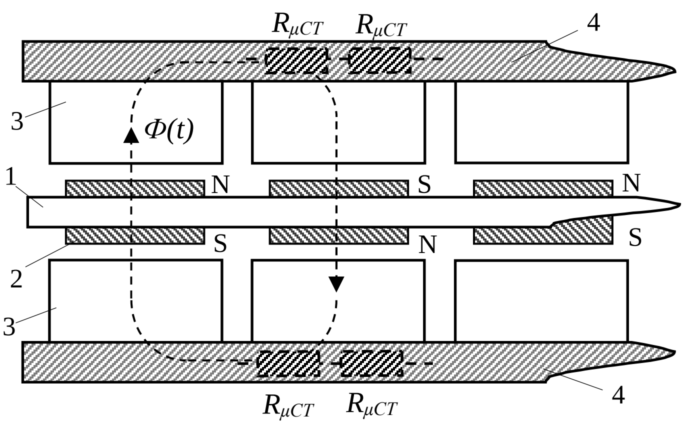

Fig. 2. Constructive scheme of part of the generator, a top view

By means of the law of full current for the m.m.s. the following expression can be obtained as an instantaneous value:

(1) $$F(t) = W.i(t) = H(t).l \ ,$$

(2) $$l = l_{в} + l_{cm} \ ,$$

where: W is the number of turns per winding;

$i(t)$ - the current through the winding;

$H(t)$ - the magnetic intensity;

l - the length of the average magnetic line of force;

$l_{в}$ - the length of the air gap;

l_{cm} - the length of the magnetic line of force through the steel stator.

By using Ohm's law for magnetic circuits, the m.m.s. is expressed again

(3) $$F(t) = R_{\mu}.\Phi(t) \ .$$

From the equality of the right members in (1) and (3), for the comprised magnetic flux it is obtained

(4) $$\Phi(t) = \frac{H(t).l}{R_{\mu}} \ .$$

From the law of electromagnetic induction the e.m.f, induced in a winding, is equal to

(5) $$e_1(t) = -W\frac{d\Phi(t)}{dt} \ .$$

Then, after substituting (4) into (5), we obtain

(6) $$e_1(t) = -\frac{W.l}{R_{\mu}}.\frac{dH(t)}{dt} \ .$$

The magnetic intensity can be expressed by means of the magnetic induction and the absolute permeability

(7) $$H(t) = \frac{B(t)}{\mu} \ .$$

From the experimental results it can be seen that the magnetic induction changes, following a sinusoidal law, and therefore it can be written as the instantaneous value B_m=1,25 T

(8) $$B(t) = B_m \sin \omega t \ .$$

By substituting (8) into (7), we obtain

(9) $$H(t)=\frac{B_m}{\mu}\sin\omega t\ .$$

Then, after substituting (9) into (6) is obtained

(10) $$e_1(t)=-\frac{W.l.B_m.\omega}{\mu.R_\mu}\cos\omega t$$

The full magnetic reluctance here is obtained from an equivalent magnetic scheme, fig. 3, and it is equal to the sum of: the air magnetic reluctances for section A - $R'_{\mu B}$, section B - $R''_{\mu B}$, and the magnetic reluctance of the steel stator $R_{\mu CT}$

(11) $$R_\mu=R'_{\mu B}+R''_{\mu B}+R_{\mu CT}\ ,$$

while the absolute permeability μ is the product of the air permeability μ_0 and the relative permeability of the corresponding material μ_r

(12) $$\mu=\mu_0.\mu_r\ .$$

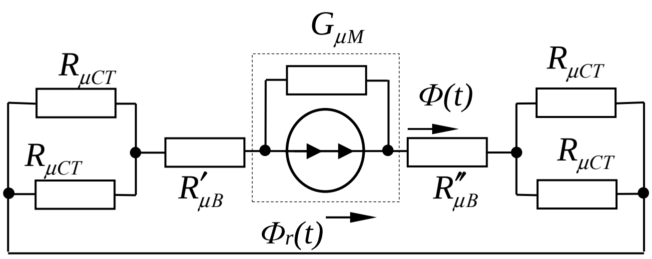

Fig. 3. Equivalent magnetic circuit

The magnetic reluctances are respectively equal to

(13) $$R'_{\mu B}=\frac{l'_B}{\mu_0.S'_B}\ ,\quad R''_{\mu B}=\frac{l''_B}{\mu_0.S''_B}\ ,\quad R_{\mu CT}=\frac{l_{CT}}{\mu_0.\mu_r.S_{CT}}\ .$$

The permanent magnet is presented by its parallel equivalent circuit, consisting of a generator of flux *$\Phi_r(t)$* and its magnetic conductivity $G_{\mu M}$

(14) $$G_{\mu M}=\frac{\mu_0.\mu_r.S_M}{l_M}\ ,$$

wherein *$\Phi(t)$*=0,8.*$\Phi_r(t)$* [6].

After substituting (11) and (13) into (10) for the amplitude value of the e.m.f. for one winding it is obtained

(15) $$E_{m1}=\frac{W.l.\omega.B_m}{\left(\frac{\mu_r l'_B}{S'_B}+\frac{\mu_r l''_B}{S''_B}+\frac{l_{CT}}{S_{CT}}\right)}\ .$$

The circular frequency ω can be expressed by means of the revolutions per minute of the generator *n* and the number of its pole pairs *p*

(16) $$\omega=\frac{2.\pi.p}{60}n\ .$$

Then, from (15) and (16) the amplitude value of the e.m.f. for one winding is equal to

(17) $$E_{m1}=\frac{W.l.B_m}{\left(\frac{\mu_r l'_B}{S'_B}+\frac{\mu_r l''_B}{S''_B}+\frac{l_{CT}}{S_{CT}}\right)}\frac{2.\pi.p}{60}n\ .$$

From (17) the effective value of the e.m.f. for one winding can be found

(18) $$E_1=\frac{W.l.B_m}{\left(\frac{\mu_r l'_B}{S'_B}+\frac{\mu_r l''_B}{S''_B}+\frac{l_{CT}}{S_{CT}}\right)}\frac{2.\pi.p}{\sqrt{2}60}n\ .$$

Since the sixteen windings in the corresponding section of the stator are connected in series, while the two sections are connected in parallel, the full r.m.s. value of the e.m.f. will be equal to

(19) $$E=\frac{16.W.l.B_m}{\left(\frac{\mu_r l'_B}{S'_B}+\frac{\mu_r l''_B}{S''_B}+\frac{l_{CT}}{S_{CT}}\right)}\frac{2.\pi.p}{\sqrt{2}.60}n\ .$$

Thus the final expression for the full r.m.s. value of the e.m.f. for the generator is obtained and in it the geometric dimensions, the parameters of the windings and the permanent magnets, as well as the number of the pole pairs of the generator and its revolutions per minute have been taken into consideration.

Investigation of the single-phase synchronous generator at idle running

By using (19), the e.m.f. at idle running can be calculated for the two sections, connected in parallel. The obtained equivalent electric circuit in a complex effective form is shown in Fig. 4.

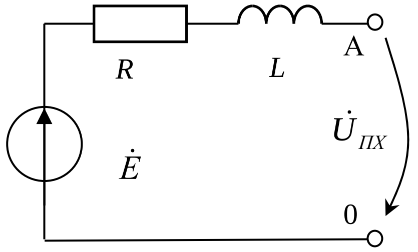

Fig. 4. Equivalent electric circuit at idle running.

In this case the r.m.s. value at idle running is equal to the phase e.m.f. of the generator

(20) $U_{ПХ}=E$.

Fig. 5 presents the phase voltage at idle running, calculated by means of the model, as well as the measured phase voltage at idle running for the prototype of the generator, both for sections A and B connected in parallel, for the studied range of revolutions per minute.

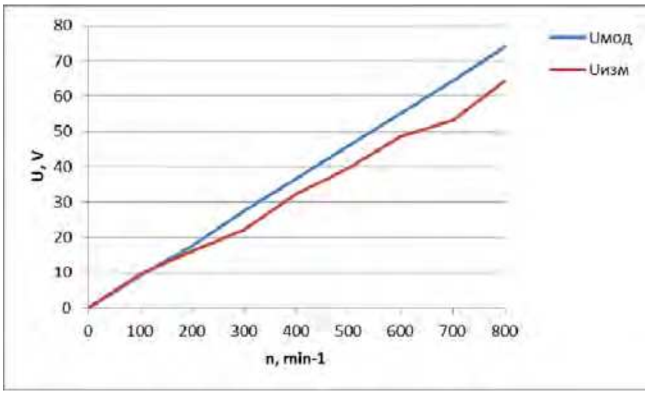

Fig. 5. Phase voltages of both the model and the generator at idle running.

The maximum relative and fiducial errors at idle running for A and B sections connected in parallel (fig. 5) are correspondingly δ_{max}=19,2% and γ_{max}=17,1%.

Investigation of the single-phase synchronous generator at active load

By using (19), the e.m.f. at active load R_T =5Ω can be calculated for both sections, when they work in parallel. In this case in an equivalent electrical circuit in a complex effective form, the active load resistance R_T is also added – fig. 7.

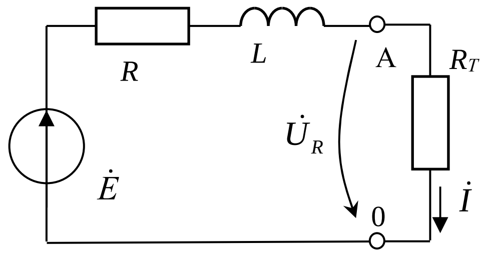

Fig. 6. Equivalent electric circuit at active load.

With the help of the second law of Kirchhoff the r.m.s. value of the phase current is found

(21) $$I=\frac{E}{\sqrt{(R+R_T)^2+(\omega.L)^2}} ,$$

where by R=2 Ω and L=10 mH the equivalent active resistance and inductance of the stator windings are denoted. Thus the r.m.s. values of the voltage on the active load and the full power can be defined

(22) $$U_R=R_T I \quad , \quad S=U_R.I \ .$$

Figures 7, 8 and 9 present the phase current, voltage and full power, calculated by means of the model, as well as those, measured for the crafted generator for working in parallel sections A and B.

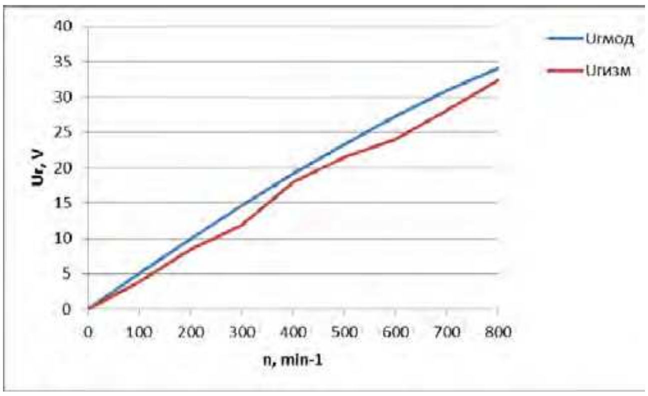

Fig. 7. Phase voltages of both the model and the generator at active load.

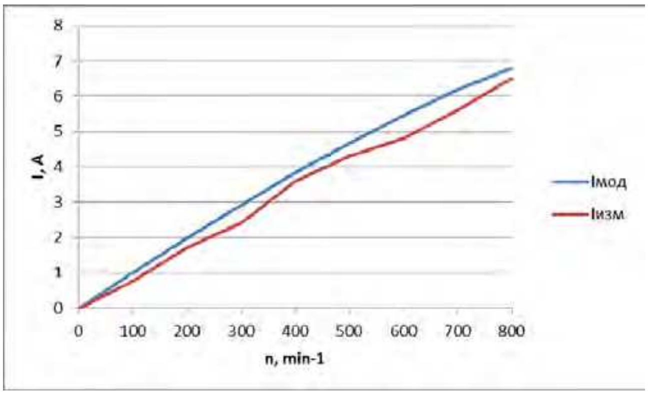

Fig. 8. Phase currents of both the model and the generator at active load.

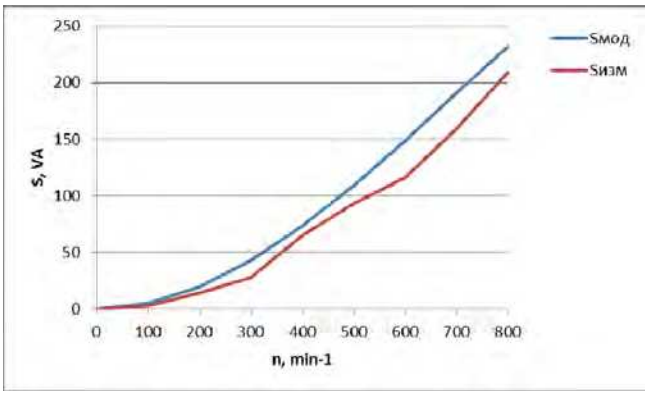

Fig. 9. Phase full power of both the model and the generator at active load.

Table 1 gives the maximum relative error and the fiducial error for the model of the generator (sections A and B connected in parallel) at active load of R_T=5Ω.

Table 1

Maximum relative and fiducial errors

	I	U_R	*S*
δ_{max}, %	10,34	13,12	18,49
γ_{max}, %	8,92	9,72	14,83

The calculated maximum relative and fiducial errors for the developed mathematical model are below 20% and in order to reduce them even more, the asymmetries should be taken into account in the process of making the generator prototype, as well as the chatter in the rotor during its rotation.

The biggest errors of the mathematical model are for the full power for the range of 600 to 700 revolutions per minute. The errors are the smallest for the phase current for sections A and B connected in parallel.

Conclusion

From the developed model and from the measurements of the single-phase synchronous 16-pole generator with axial magnetic field and rare earth magnets the following basic conclusions can be drawn up:

1. In the obtained expression for the full r.m.s. value of the e.m.f., the following parameters have been taken into account: the geometric dimensions, the characteristics of the windings and the permanent magnets, the number of the pole pairs of the generator, and its revolutions per minute.
2. The obtained mathematical model simulates with good precision the work of the studied prototype of a generator, wherein the maximum relative error and the fiducial error are below 20%.
3. In order to reduce the maximum relative error, as well as the fiducial error, it is necessary to take into consideration the asymmetries in the process of making the prototype of the generator, as well as the chatter in the rotor during its rotation.
4. The biggest errors for the model are observed for the full power in the range from 600 to 700 revolutions per minute.
5. The phase currents, voltages and full power, obtained by means of the model, are always higher than the corresponding measurements for the crafted generator, what is due to the asymmetries in the location of the magnets in the rotor and of the windings in the stator.
6. The phase current, voltage and full power, calculated by means of the model, and the ones, measured for the crafted generator, depend almost linearly on the revolutions per minute.
7. When the air gap is reduced, part of the magnet reluctances decrease and this could improve the output electrical parameters of the generator.
8. The magnetic induction changes, following a sinusoidal law, what is experimentally established by measuring the output voltages by an oscilloscope both at idle running and at active load.

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Associate Professor Nikola Georgiev, PhD, *is a lecturer at Sofia Technical University – Plovdiv Branch. He graduated from HIMEE-Gabrovo in 1986. He works in the field of sensors of non-electrical quantities and wind generators for power up to 1 kW.*

tel.: 032 659 592, e-mail: geotek@abv.bg

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