

Model order reduction for a single machine infinite bus power system

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The paper considers the problem of model order reduction for a single machine infinite bus power system by using a Legendre polynomials approximation based balanced residualization method. The model examines the small signal behavior and explores the existence of dynamic stability for a synchronous generator connected to a large system through transmission lines. The single machine infinite bus is a linearized power system model consisting of two inputs, two outputs and ten state variables. The goal for model reduction is to facilitate the excitation, the governor controller and the stabilizing circuits design and to reduce the computational complexity in exploring the synchronous machine. The proposed method for model reduction is a two stage method, where a balanced realization is initially obtained and then, it is subjected to a singular perturbation approximation. Legendre polynomials approximation for the system gramians is used to avoid solving the usual large-scale Lyapunov equations and thus, reducing the complexity of the computational procedure. Several numerical experiments are performed, showing the good approximation properties of the presented method.

Редуциране на реда на модела на генератор свързан към енергийната мрежа (Камен Л. Перев) Тази статия разглежда задачата за редуциране на реда на модела на синхронен генератор свързан към енергийната мрежа по метода на балансовата резидуализация с използване на полиномна апроксимация по Лежандър. Разгледаният модел изследва поведението на системата при малки сигнали и изучава динамичната устойчивост на синхронния генератор. Моделът представлява линеаризирано математично описание на електрическата система, което се състои от два входа, два изхода и десет променливи на състоянието. Целта е да се опрости синтеза на веригата на възбуждане, регулатора и стабилизиращите устройства, както и да се намали изчислителната сложност при изследване на синхронната машина. Предложеният метод се състои от две стъпки: при първата стъпка се генерира балансова реализация на първоначалния модел, а при втората стъпка се изчислява апроксимация по метода на сингулярните смущения. В изчислителната процедура грамианите на системата се апроксимират в ортогонален ред на Лежандър, като по този начин се избягва решаването на обичайните уравнения на Ляпунов. Проведени са няколко експеримента, които потвърждават добрите апроксимиращи свойства на метода.

1. Introduction

Dynamical system modeling is concerned with describing the relations and interconnections characterizing the explored physical processes. There always exists a trade-off between the complexity of the model and its accuracy. As higher is the accuracy of the mathematical description, as larger is the number of the differential equations representing this description. In such cases it is desirable to reduce the equations number while preserving the key features of the examined physical phenomenon. The procedure of model simplification by decreasing the dimensionality of the primary model is called model order reduction.

Model order reduction finds application in simulation of physical processes and for implementation of lower-order controllers in feedback systems.

Parallel with increasing the complexity of the system model, increases the complexity of the corresponding system controllers. Implementation of high-order controllers is usually accompanied with high computational cost, low reliability and different maintenance problems [1]. There exist three main approaches in reducing the size of the controller [2]: *i*) the direct methods, where low-order controllers are directly designed for high-order plants, *ii*) the indirect methods, where for high-order plants we design first

high-order controllers and then reduce the size of the controller and finally *iii*) we approximate the plant rather than the controller, where for high-order plants reduce first the order of the plant and then design the corresponding low-order controller. Model reduction is generally characterized with reducing the number of the state variables in the system description. There exist two main approaches for reducing the size of the state vector in control systems: state vector truncation and residualization [3]. State truncation is related with elimination of part of the state vector in the procedure of model reduction and the system matrices are obtained by truncation of columns and rows corresponding to the eliminated state vector components. If the reduced order model is obtained by balanced truncation, it preserves stability and enjoys a predetermined bound on the error of approximation. The transfer function of the reduced order system at infinity is equal to the transfer function of the original system at infinity however, the DC gains of the high order and the reduced order models mismatch [3, 4]. An alternative to state truncation is state residualization. State residualization is based on the method of singular perturbations, where the state vector is partitioned on its slow and fast part. The system model is reduced by neglecting the derivative of the state vector fast components and solving the obtained algebraic equation with respect to these state variables. The residualization method preserves the steady state gain of the system and possesses good accuracy in the low frequency range [3].

The problem of model reduction for a single machine infinite bus power system has been explored since the development of the small-signal model in [5]. In [6] is presented a mixed model reduction technique, which is based on partial fraction expansion together with the dominant poles concept of the power system. The obtained procedure is frequency domain based and accounts for certain approximation errors in steady state. However, the method fails to give clear relation between the order of truncation and the error of approximation. Genetic algorithm is used in [7] and a particle swarm algorithm is used in [8], where the goal is to design thyristor controlled series compensators for power oscillation damping and improving the power system stability properties. In both cases the direct approach for low-order controller design is utilized and the convergence properties of the proposed algorithms are explored. Both algorithms are heuristic in nature; they succeed in a certain degree for improving stability and reducing power oscillations however, fail to quantify the level of achieved performance.

The proposed method is indirect in nature and achieves model order reduction for the single machine infinite bus power system. It combines the singular perturbation approximation technique with balanced truncation of a minimal stable system. The method is based on performing residualization over already balanced realization, where the balancing transformation is obtained by orthogonal polynomial approximation of system gramians. The computational algorithm is based on collecting system trajectories data, where a data snapshots matrix is utilized to compute the Fourier coefficients vectors for a Legendre polynomial series approximation of the system state impulse response. The obtained Fourier coefficients vectors are used for approximation of the reachability and observability gramians and thus, to avoid solving Lyapunov equations for computation of the balancing transformations. The proposed method preserves stability of the reduced order system, supports an easily calculable error bound and assures small approximation error in the low as well as in the high frequency ranges.

2. Legendre polynomials approximation based balanced realization and singular perturbation approximation of stable linear systems

Consider the stable linear time-invariant system described by its state space model:

$$(1.1) \quad \dot{x}(t) = Ax(t) + Bu(t), \quad t \geq 0$$

$$(1.2) \quad y(t) = Cx(t) + Du(t), \quad x(0) = x_0,$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$. The reachability gramian on the interval $[0, T]$ is given by the expression:

$$(2) \quad W_r(0, T) = \int_0^T e^{At} BB^T e^{A^T t} dt.$$

The observability gramian on the interval $[0, T]$ is defined as follows:

$$(3) \quad W_o(0, T) = \int_0^T e^{A^T t} C^T C e^{At} dt.$$

The gramians are essential system characteristics and have an important energy interpretation. The smallest amount of energy needed to move the system from zero to the state x is given by the expression $E_r = x^T W_r^{-1} x$, while the energy obtained by observing the output of the system with initial condition x and no input function is given by the expression $E_o = x^T W_o x$. The states that correspond to small singular values of the reachability gramian are difficult to reach and the states corresponding to small singular values of the observability gramian are

difficult to observe. Balancing is the process of transforming the system states into a coordinate system, where the reachability and observability gramians are equal diagonal matrices, i.e.:

$$(4) \quad W_r(0, T) = W_o(0, T) = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_n)$$

with $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n$, where σ_i , $i=1, 2, \dots, n$ are the Hankel singular values. These states which are difficult to reach are also difficult to observe. Truncating these states will have limited effect on the input/output behavior of the system and will result to a reduced order system which preserves stability and the error of approximation is bounded by the following expression:

$$(5) \quad \|G(j\omega) - G_k(j\omega)\|_\infty \leq 2(\sigma_{k+1} + \dots + \sigma_n),$$

where k is the order of the reduced system. Based on the truncated Hankel singular values the already balanced system can be partitioned as follows:

$$(6) \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C = [C_1 \quad C_2]$$

and $\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$, $\Sigma_1 = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_k)$ and

$\Sigma_2 = \text{diag}(\sigma_{k+1}, \sigma_{k+2}, \dots, \sigma_n)$. Next we partition the state vector in correspondence to the matrix partitioning in (6) as $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. When x_2 is a stable

state vector, the real parts of the eigenvalues of A_{22} are negative, $\text{Re}\{\lambda_i(A_{22})\} < 0$, $i=1, 2, \dots, n-k$.

Then setting \dot{x}_2 equal to zero we obtain the following system:

$$(7.1) \quad \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u,$$

$$(7.2) \quad 0 = A_{21}x_1 + A_{22}x_2 + B_2u,$$

$$(7.3) \quad y = C_1x_1 + C_2x_2 + Du.$$

Solving equation (7.2) with respect to the state variables x_2 we obtain the following realization:

$$(8.1) \quad \dot{x}_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_1 + (B_1 - A_{12}A_{22}^{-1}B_2)u,$$

$$(8.2) \quad y = (C_1 - C_2A_{22}^{-1}A_{21})x_1 + (D - C_2A_{22}^{-1}B_2)u.$$

The above system presentation is realizable because the state variables x_2 are stable and hence matrix A_{22} is invertible. The system (8) is called balanced residualization of the original system [9]. The balanced residualization system is a slow singular perturbation approximation of the balanced system if matrix A_{22} is stable and its eigenvalues are fast. Since the balanced residualization system is obtained by applying balancing transformation, the error of

reduction at very high frequencies tends to zero [4]. Since the singular perturbation method is used for removing the fast state variable, the balanced residualization system preserves the DC gain [9]. Therefore, the balanced residualization system has small approximation error at low, as well as at high frequencies. The stability of the original system allows to use the gramians at infinity, i.e. $W_r = \lim_{T \rightarrow \infty} W_r(0, T)$ and $W_o = \lim_{T \rightarrow \infty} W_o(0, T)$ and then to

solve certain Lyapunov equations for their computation. The proposed method avoids solving the Lyapunov equations for computing the gramians, but instead relies on the information obtained from the trajectories of the system to approximate the gramians. The reachability and observability gramians are approximated by using an orthogonal series expansion of certain impulse response characteristics. The method is more practical in sense that the trajectories data can be obtained either by measurement or by using computer simulations. The following derivations present a short introduction to the approximation procedure, which is described in detail in [10]. Consider the state response of system (1) obtained by solving the state equation:

$$(9) \quad x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau.$$

We assume zero initial conditions $x_0 = 0$. Consider the single-input single-output case first when $B = b$ is a vector column and $C = c$ is a vector row. If we apply at the input a delta impulse $u(t) = \delta(t)$, then the state impulse response is obtained in the form $x(t) = e^{At}b$. Therefore, the reachability gramian can be obtained as:

$$(10) \quad W_r(0, T) = \int_0^T e^{At}bb^T e^{A^T t} dt = \int_0^T x(t)x^T(t)dt.$$

The Legendre orthogonal polynomials form a complete set of orthogonal polynomials in the Hilbert space $L_2[-1, 1]$ with weighting function $w(t) = 1$, $t \in [-1, 1]$. The Legendre polynomials satisfy the recurrence relation [11]:

$$(11) \quad P_{n+1}(t) = \frac{(2n+1)tP_n(t) - nP_{n-1}(t)}{n+1},$$

$P_0(t) = 1$, $P_1(t) = t$, $n = 1, 2, \dots$. The Legendre polynomials can be normalized with a constant to obtain the Legendre orthonormal functions

$$\varphi_n = \sqrt{\frac{2n+1}{2}}P_n.$$

Very often the definition interval of the approximated function is bounded but with different boundary points. For example, a complete set

of Legendre orthonormal functions in the vector space $L_2[0, T]$ is obtained by introducing a change of variables as follows:

$$(12) \quad \varphi_n(\tau) = \sqrt{\frac{2n+1}{2}} P_n(\tau), \quad \tau = \frac{2}{T}t - 1, \quad t \in [0, T].$$

Then a series representation of the function $q(t) \in L_2[0, T]$ takes the following form:

$$(13) \quad q(t) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2n+1}{2}} P_n\left(\frac{2}{T}t - 1\right), \quad t \in [0, T],$$

$$(14) \quad c_n = \sqrt{\frac{2n+1}{2}} \int_0^T q(t) P_n\left(\frac{2}{T}t - 1\right) dt.$$

Since $x(t) \in L_2[0, T]$ is the state impulse response for a linear stable system, it assumes Legendre series representation on the interval $[0, T]$ as follows:

$$(15) \quad x(t) = \sum_{n=1}^{\infty} f_n \sqrt{\frac{2n+1}{2}} P_n\left(\frac{2}{T}t - 1\right) = \sum_{n=1}^{\infty} f_n \varphi_n\left(\frac{2}{T}t - 1\right)$$

where f_n , $n=1, 2, \dots$ are the Fourier coefficients in the Legendre series representation which can be computed by (14). Therefore, the reachability gramian can be approximated by the expression:

$$(16) \quad W_r(0, T) = \int_0^T x(t)x^T(t) dt \\ \approx \frac{T}{2} \int_0^T \sum_{k=0}^N f_k \varphi_k\left(\frac{2}{T}t - 1\right) \sum_{k=0}^N f_k^T \varphi_k\left(\frac{2}{T}t - 1\right) dt = \frac{T}{2} \sum_{k=0}^N f_k f_k^T$$

where N is the order of truncation of the Legendre series representation and the coefficient $T/2$ appears after change of variables in integration. For computing the observability gramian consider the linear time-invariant dual system representation in the form:

$$(17.1) \quad \dot{p}(t) = A^T p(t) + C^T \tilde{u}(t),$$

$$(17.2) \quad \tilde{y}(t) = B^T p(t) + D \tilde{u}(t).$$

The observability gramian of the original system (1) is equal to the reachability gramian of the dual system (17). By approximating the state impulse response of the dual system in terms of Legendre orthogonal functions in the form $p(t) = \sum_{n=1}^{\infty} g_n \varphi_n\left(\frac{2}{T}t - 1\right)$, we

can compute a Legendre series approximation of the observability gramian of system (1) as follows:

$$(18) \quad W_o(0, T) = \int_0^T p(t)p^T(t) dt \\ \approx \frac{T}{2} \int_0^T \sum_{k=0}^M g_k \varphi_k\left(\frac{2}{T}t - 1\right) \sum_{k=0}^M g_k^T \varphi_k\left(\frac{2}{T}t - 1\right) dt = \frac{T}{2} \sum_{k=0}^M g_k g_k^T$$

where M is the order of truncation of the Legendre series representation. For the multi-input multi-output

$[m \times p]$ system the reachability gramian can be approximated by $W_r(0, T) \approx \frac{T}{2} \sum_{k=0}^N \sum_{i=1}^m f_k^i (f_k^i)^T$, where

f_k^i are the vector Fourier coefficients when the input signal is $\delta(t)e_i$ and e_i is the i -th column of the identity matrix I_m , $i=1, 2, \dots, m$. The corresponding observability gramian is approximated by $W_o(0, T) \approx \frac{T}{2} \sum_{k=0}^N \sum_{j=1}^p g_k^j (g_k^j)^T$, where g_k^j are the

vector Fourier coefficients when the input signal for the dual system is $\delta(t)e_j$ and e_j is the j -th column of the identity matrix I_p , $j=1, 2, \dots, p$. Then, the

balancing transformation for the stable linear time-invariant system (1) can be computed by the square root algorithm with the following steps: *i*) perform a Cholesky decomposition on the reachability gramian $W_r(0, T) = UU^T$, *ii*) perform a Cholesky decomposition on the observability gramian $W_o(0, T) = LL^T$, *iii*) perform singular value decomposition on the product of Cholesky factors $U^T L = W \Sigma V^T$, *iv*) compute the similarity transformation matrices for the change of basis procedure $P = \Sigma^{-1/2} V^T L^T$ and $P^{-1} = U W \Sigma^{-1/2}$, *v*) apply the similarity transformation to system matrices and obtain the system model into a balanced form $A_b = P A P^{-1}$, $B_b = P B$, $C_b = C P^{-1}$ and $D_b = D$.

3. Mathematical description of the single machine infinite bus power system

In this section we describe a synchronous generator delivering power through a step-up transformer and a high voltage transmission line to an infinite grid, see Fig.1. X_T and X_L represent the transformer and transmission line reactances, respectively and R_L is the transmission line resistance. Generator terminal and infinite bus voltages are represented by V_T and V_B . Because of the relative size of the system to which the machine is supplying power, dynamics associated with the machine will cause virtually no change in the magnitude and frequency of the voltage V_B . Such a voltage source of constant magnitude and constant frequency is referred to as an infinite bus [12]. The magnitude of the infinite bus voltage V_B remains constant when the machine is perturbed. However, as the steady state conditions change, the magnitude of

V_B may change due to the changing conditions of the external network.

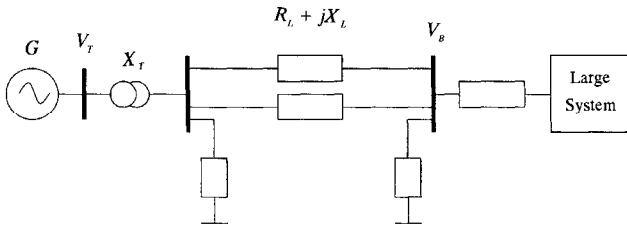


Fig.1. Synchronous generator connected to a large system

The small signal behaviour of the single machine infinite bus power system is initially described by the classical linearized model developed in [5] and finds its detailed characterization in [12]. The transfer function approach is first used by Heffron and Philips to analyze the small signal stability of synchronous machines. The block diagram of the three-phase two-pole 160 MVA turbogenerator with an automatic excitation control system consisting of a field circuit exciter, rate feedback and power system stabilizer is shown in Fig.2.

The linearization is performed about initial operating conditions represented by the base rotor angle $\delta = \delta_0$, the base rotor electrical speed $\omega_r = \omega_0$, the mutual flux linkage $\psi_{fd} = \psi_{fd,0}$, the field voltage $E_{FD} = E_{FD,0}$, the terminal voltage $V_t = V_{t,0}$ and the mechanical torque $T_m = T_{m,0}$. Therefore, all the physical variables are deviations from the operating point system variables. For simplicity of notation the deviation signals will be represented by their actual symbols.

For example the rotor angle deviation $\Delta\delta$ will be represented by the variable δ and the per unit speed deviation $\Delta\omega_r$, where $\Delta\omega_r = \frac{(\omega_r - \omega_0)}{\omega_0}$ will be

represented by the variable ω_r . What follows is a brief description of the main building blocks of the single machine infinite bus power system. More complete description can be found in [12]. In Fig.2 the synchronous generator classical model is represented by its mechanical and electrical loops, where T_a , T_e and T_m are the accelerating, air-gap and mechanical torques, H is the inertia constant, K_s is the synchronizing torque coefficient and K_D is the damping coefficient. The generator model inputs are the mechanical torque T_m and the field flux linkage voltage E_q and the model outputs are the base rotor

angle δ and the generator terminal voltage V_t . K_1 , K_4 , K_5 and K_6 are linear model parameters of the synchronous generator. The field circuit of the electrical machine has as a reference signal the exciter output voltage E_{FD} and as an output signal the voltage E_q proportional to the d -axis field flux linkage ψ_{fd} . K_3 is the gain and T_3 is the time constant of the field circuit. The exciter system has as input signal the regulator output voltage V_R and its parameters are the self-excited field constant K_E , the saturation function parameter S_E and the exciter time constant τ_E . The saturation function parameter S_E is the gain of the feedback around the exciter and serves the goal to avoid saturation at the output. The voltage regulator parameters are the regulator gain K_A and the regulator time constant τ_A . The goal of the exciter and the voltage regulator is to provide the required synchronizing torque and transient response stability. The rate feedback serves the role to stabilize the field voltage, which is the output of the exciter system. Its parameters are the rate feedback gain K_F and the rate feedback time constant τ_F . The voltage transducer circuit relates the generator terminal voltage V_t to the input of the voltage regulator. K_R is the transducer gain and τ_R is the terminal voltage transducer time constant.

An effective way to meet the conflicting exciter performance requirements with regard to system stability is to provide a power system stabilizer (PSS). The basic function of the power system stabilizer is to add damping to the generator rotor oscillations [12]. Since the role of the PSS is to introduce more torque damping it is natural to use as input signal the angular velocity signal ω_r . The PSS circuit consists of three parts: a phase compensation circuit, a signal washout circuit and a gain. The phase compensation circuit with time constants τ_1 , τ_2 , τ_3 and τ_4 has a phase-lead characteristic to compensate the phase lag between the exciter and the electrical air-gap torque. The signal washout circuit serves as a high-pass filter with time constant τ_0 and allows the PSS to respond only to changes in speed but not in steady values of the angular velocity deviation. The stabilizer gain K_0 determines the amount of damping in the angular velocity.

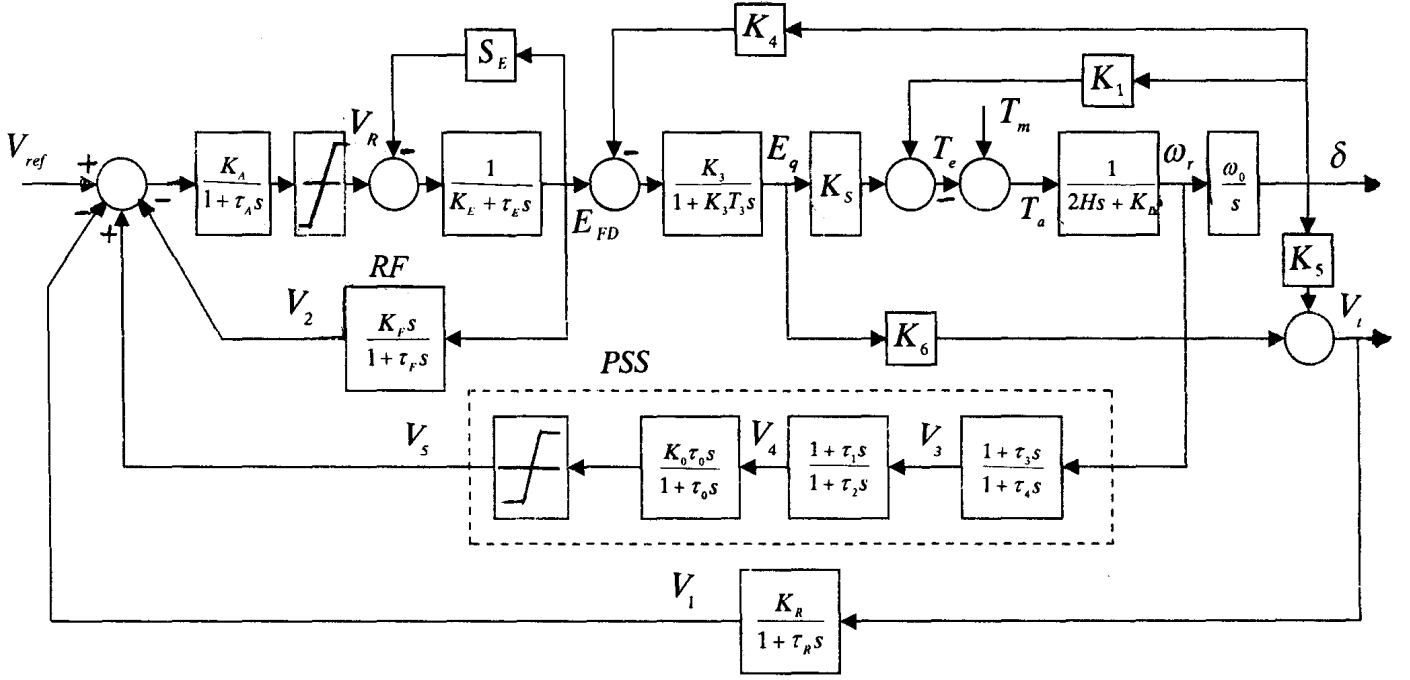


Fig.2. Single machine infinite bus power system block diagram

4. Numerical experimental results

The state space model of the single machine infinite bus power system described by equations (1) has two input signals $u_1 = V_{ref}$ and $u_2 = T_m$, two output signals $y_1 = \delta$ and $y_2 = V_t$ and ten state variables: $x_1 = E_q$, $x_2 = \omega_r$, $x_3 = \delta$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$, $x_7 = V_4$, $x_8 = V_5$, $x_9 = V_R$ and $x_{10} = E_{FD}$. The system matrices are given as follows [6]:

$$A = \begin{bmatrix} -0.55 & 0 & -0.31 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.17 \\ -0.04 & 0 & -0.04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 314.16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9.55 & 0 & -0.87 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0.04 & -0.03 \\ -0.2 & 10.87 & -0.17 & 0 & 0 & -10.87 & 0 & 0 & 0 & 0 & 0 \\ -0.94 & 51.98 & -0.8 & 0 & 0 & -41.12 & -10.87 & 0 & 0 & 0 & 0 \\ -0.94 & 51.98 & -0.8 & 0 & 0 & -41.11 & -10.87 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1000 & -1000 & 0 & 0 & 0 & 1000 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.05 & -0.82 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0.09 & 0 & 0 & 0 & 0.44 & 2.12 & 2.12 & 0 & 0 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.48 & 0 & -0.04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Obviously, this is a linear multivariable time-invariant stable system model with ten state variables, two inputs and two outputs. The Hankel singular

values which are obtained by applying a Legendre series approximation of system gramians with order of series truncation $N = 35$ are computed as follows:

$$\Sigma = \begin{bmatrix} 12.17 & 10.67 & 2.5 & 1.06 & 0.14 & 0.6 \cdot 10^{-2} \\ 0.2 \cdot 10^{-2} & 0.72 \cdot 10^{-3} & 0.19 \cdot 10^{-3} & 0.7 \cdot 10^{-4} & & \end{bmatrix}.$$

The error between the full order system model and the reduced order models is governed by the relation (5). Looking at matrix Σ we can observe that the last five Hankel singular values are considerably small and we can expect that reducing the system model with five state variables corresponding to these Hankel singular values will not result to a significant change in the input/output behavior of the power system. Further reducing the order of the system model will introduce more error, but we still anticipate that the reduced third order model will be quite accurate.

The simulations are performed on the time interval $[0, T]$, where $T = 20s$ with discretization step $\Delta = 5ms$. Fig. 3 presents the unit step responses of the single machine infinite bus power system with respect to the input signal $u_1 = V_{ref}$ (when the signal $u_2 = T_m = 0$) and the output signal $y_1 = \delta$ for the full order model and the corresponding reduced fifth and third order models. It can be clearly seen that the step response of the fifth order system model almost coincides with the step response of the full order model. Some more deviation appears between the step responses of the full order model and the reduced third order model.

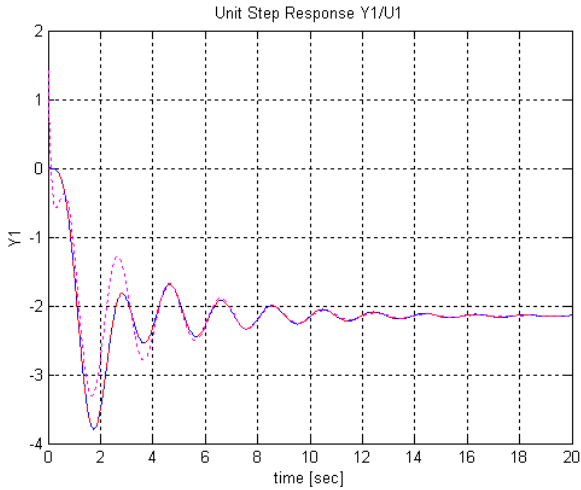


Fig.3. Unit step response $y_1 = \delta$ vs. $u_1 = V_{ref}$
full order----, reduced 5th -.-.-, reduced 3rd...

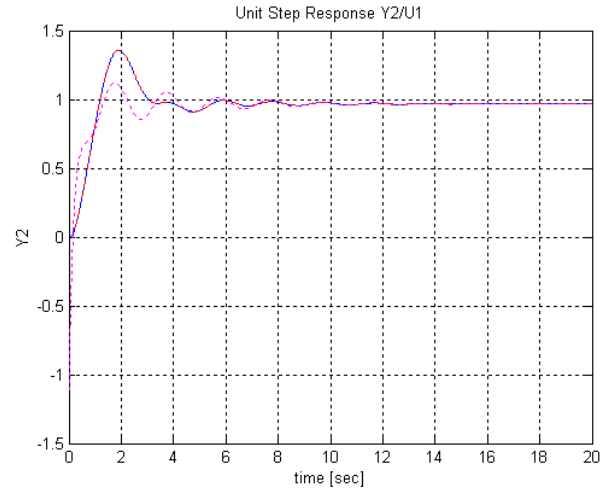


Fig.5. Unit step response $y_2 = V_t$ vs. $u_1 = V_{ref}$
full order----, reduced 5th -.-.-, reduced 3rd...

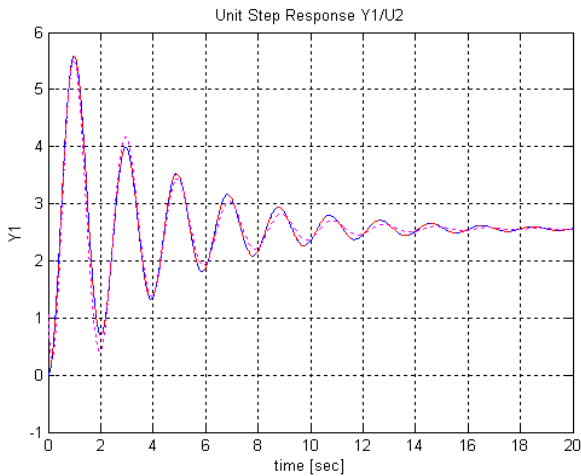


Fig.4. Unit step response $y_1 = \delta$ vs. $u_2 = T_m$
full order----, reduced 5th -.-.-, reduced 3rd....

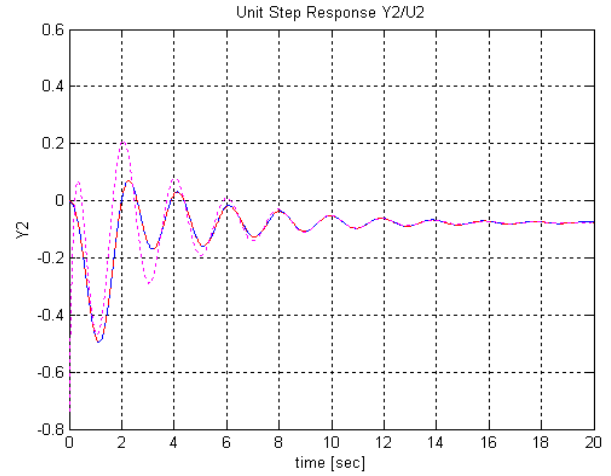


Fig.6. Unit step response $y_2 = V_t$ vs. $u_2 = T_m$
full order----, reduced 5th -.-.-, reduced 3rd...

The relative error norms between the outputs $y_1 = \delta$ of the full order model and the reduced fifth and third order models when the input signal is $u_1 = V_{ref}$ ($u_2 = T_m = 0$) are presented as a quantitative measure for the approximation power of the method:

$$\frac{\|y_1 - y_{1,5}\|_2}{\|y_1\|_2} = 1.969 \cdot 10^{-4}, \quad \frac{\|y_1 - y_{1,3}\|_2}{\|y_1\|_2} = 0.0971 \cdot$$

Fig. 4 presents the unit step responses of the output $y_1 = \delta$ with respect to the input signal $u_2 = T_m$ when $u_1 = V_{ref} = 0$ of the full order model and the reduced fifth and third order system models. It can be clearly seen that all three responses are similar and match very closely. The relative error norms are computed as follows:

$$\frac{\|y_1 - y_{1,5}\|_2}{\|y_1\|_2} = 2.189 \cdot 10^{-4}, \quad \frac{\|y_1 - y_{1,3}\|_2}{\|y_1\|_2} = 0.0483$$

Fig. 5 presents the unit step responses of the single machine infinite bus power system with respect to the input signal $u_1 = V_{ref}$ (when the signal $u_2 = T_m = 0$) and the output signal $y_2 = V_t$ for the full order model and the corresponding reduced fifth and third order models. It is clearly seen that the full order model and the fifth reduced order model step responses are very close and the error is insignificant. However, there is a definite difference between the step responses of the full order model and the corresponding reduced third order model. The quantitative measures are presented by the relative error norms which are computed as follows:

$$\frac{\|y_2 - y_{2,5}\|_2}{\|y_2\|_2} = 3.372 \cdot 10^{-4}, \quad \frac{\|y_2 - y_{2,3}\|_2}{\|y_2\|_2} = 0.1063 \cdot$$

The unit step responses of the single machine infinite bus power system with respect to the input signal $u_2 = T_m$ (when the signal $u_1 = V_{ref} = 0$) and the output signal $y_2 = V_r$ for the full order model and the corresponding reduced fifth and third order models are shown in Fig.6. It is evident again the small error between the step responses of the full order and the reduced fifth order models. Much larger error appears for the step response of the third order model which is demonstrated by the relative error norms:

$$\frac{\|y_2 - y_{2,5}\|_2}{\|y_2\|_2} = 0.0029, \quad \frac{\|y_2 - y_{2,3}\|_2}{\|y_2\|_2} = 0.5258 \cdot$$

The obtained results are expected from (5) and the error characterization between the full order and reduced order models confirms the influence of the Hankel singular values presented in matrix Σ on the approximation properties of the proposed method.

5. Conclusion

The paper considers the problem of model order reduction of a power system by applying a Legendre polynomials approximation based balanced residualization method. The proposed method possesses good approximation properties in the low as well as in the high frequency ranges. The model reduction procedure is trajectories based, it is implemented in deterministic settings and the input data for the computational algorithm can be obtained either from measurement or from simulation. Legendre polynomials series is used to obtain the reachability and observability gramian approximations and thus, the process of solving the usual Lyapunov equations is avoided. The suggested method is applied to reduce the order of the single machine infinite bus power system linearized model. Several experiments are performed comparing the step responses of the full order model and the reduced fifth and third order models obtained by balanced residualization. The experiments confirm the good approximation capabilities of the proposed method by comparing as a quantitative measure the computed relative error norms of the power system output variables between the full order and the corresponding reduced order models.

REFERENCES

- [1] Gu, D.-W., P. Petkov, M. Konstantinov. Robust control design with MATLAB. Springer-Verlag, London, 2005.
- [2] Anderson, B.D., Y. Liu. Controller reduction: Concepts and approaches. IEEE Trans. Automat. Control, vol. 34, 8, 1989, pp. 802 – 812.
- [3] Antoulas, A. Approximation of large – scale dynamical systems. SIAM Publ., Philadelphia, 2005.
- [4] Liu Y., B.D. Anderson. Singular perturbation approximation of balanced systems. Int. J. Control, vol. 50, 4, 1989, pp. 1379 – 1405.
- [5] Heffron, W.G., R.A. Phillips. Effect of modern amplitudyne voltage regulators on underexcited operation of large turbine generators. AIEE Trans., vol. PAS-71, 1952, pp. 692 – 697.
- [6] Papadopoulos, D. P. Order reduction of linear system models with a time-frequency domain method. J. Franklin Inst., vol. 322, 4, 1986, pp. 209 – 220.
- [7] Hadi, S.P., H.I. Wiennetou R.F. Mochamad. TCSC power oscillation damping and PSS design using genetic algorithm modal optimal control. Int. J. Eng. Comp. Science, vol. 13, 1, 2013, pp. 23 – 30.
- [8] Yazdekhasti, A., I. Sadeghkhani. Optimal tuning of TCSC controller using particle swarm optimization. Adv. Electr. Engin. System, vol. 1, 1, 2012, pp. 24 – 29.
- [9] Samar, R., I. Postlethwaite, D.-W. Gu. Model reduction with balanced realizations. Int. J. Control, vol. 62, 1, 1995, pp. 33 – 64.
- [10] Perv, K. Balanced truncation by using Legendre orthogonal polynomial approximation. Engineering Sciences, 3, 2014, pp. 5 – 21 [in Bulgarian].
- [11] Abramowitz, M., I. Stegun, eds. Handbook of mathematical functions with formulas, graphs and mathematical tables. Dover Publ., N. Y., 1972
- [12] Kundur, P. Power system stability and control. McGraw-Hill Inc., N.Y., 1993

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