Network topology analysis - graph properties that matter

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This article presents correlation between network metrics with graph properties, more specifically spectral graph characteristics. This work is in the foundation of network topology analysis based on the relevant graph representation of the network and its properties. In the beginning is made an overview of the basic network metrics that contribute to the network quality, resilience, robustness and performance, which in turn reflects in the satisfaction of the final user, which is usually measured with the Quality of Experience (QoE) level. These metrics include, but are not limited to: • Link latency and Round-trip time • Jitter • Throughput • Packet loss • Availability. The next part is focus on topology parameters of graphs, how they are calculated and what their meaning in the context of networks is. Focus is given on the so called spectral graph properties, because they well represent global metrics of the topology in terms of quality and resilience. Such spectral graph properties are the algebraic connectivity, graph spectrum and some of their products such as graph diameter, effective resistance and network criticality. In the third part is given a view on the correlation between the network metrics and graph properties, so the analysis of the properties of the graph that represents the real network, can give pretty good estimation on the network quality in total. Based on this analysis, the graph topology can be optimized, so that the network that it represents can be also optimized to achieve better quality. Such network optimization ultimately leads to improving the QoE level of a network.

Анализ на топологичната структура на мрежата – свойства на графите, които имат значение (Мирчо Мирчев, Сеферин Мирчев). *Тази статия представя корелация между мрежовите показатели чрез свойства на графите и по-специално чрез спектралните характеристики на графите. Работата е в основата на анализа на топологичната структура на мрежата въз основа на съответното представяне на мрежата и нейните свойства чрез графи. В началото е направен преглед на основните показатели на мрежата, които допринасят за качеството на мрежата, гъвкавостта, устойчивостта и производителността, което от своя страна отразява удовлетворението на крайния потребител, което обикновено се измерва с нивото на качество на възприемане. Тези показатели включват, но не се ограничават до: закъснения и време за изпращане и връщане на пакета, джитер, пропускателна способност, загуба на пакети, наличност. Следващата част се фокусира върху топологичните параметри на графите, как те се изчисляват и какъв е техният смисъл в контекста на мрежи. Набляга се върху така наречените спектрални свойства на графите, тъй като те добре представят глобални показатели на топологичната структура по отношение на качество и гъвкавост. Такива свойства на спектралните графи са алгебричната свързаност, спектър на графа и някои от техните производни като диаметър на графа, ефективна устойчивост и критичност на мрежата. В третата част е даден поглед върху корелацията между мрежовите показатели и свойствата на графа, така че анализът на свойствата на графа, който представя реалната мрежа, да може да даде доста добра оценка за качеството на мрежата като цяло. Въз основа на този анализ, топологичната структура на графа може да бъде оптимизирана, така че мрежата, която представлява, също да бъде оптимизирана за постигане на по-добро качество. Такава мрежова оптимизация в крайна сметка води до подобряване на нивото на възприемане на мрежата.*

I. Introduction

The quick expansion of networks worldwide, leads to number of question regarding their topology. In the initial planning and further optimizations the network topology is a key factor. The dynamic routing protocols give the ability to have global networks implementing a variety of complex topologies. Usually there are constrains for choosing a network topology, which are geographical, economical and technical. However often disregarded questions is about now to choose a topology – Which one to choose initially?, How the ensure the needed redundancy and resilience of the network?, etc. Choosing the "right" topology has direct effect on the overall network performance and quality and it contributes to the end-user experience. This in turn corresponds to the satisfaction of the final user, which is usually measured with the Quality of Experience (QoE) level.[1] Although QoE parameters are usually based on subjective measures that rely on human opinion, most of the network and Quality of Service (QoS) metrics, which are strictly related to technical measurement over the network, affect the QoE parameters. Several approaches are taken to measure and optimize the QoE of networks [1]–[7]. Some, like [4], propose deploying an overlay layer to the physical network that can cope with link failures and congestion. But again stays the question about the topology of these networks.

On the other hand every network can be represented as a graph, where network nodes are the vertices and links – edges. As there are a lot of studies on graph properties, these can be easily evaluated and measured [8]–[11].

Most of the graph studies focus on the graph properties per se, and don't relate them to the network metrics. This paper presents correlation between network metrics with graph properties, more specifically spectral graph characteristics [8], [11]–[17]. This correlation is important because there are very strong analytical methods for graph analysis and optimization and having this correlation can give possibility to optimize the network metrics by optimizing the graph properties [18].

II. Network metrics

Today's networks are almost all packet based, more specifically IP based. Therefore for all further references to network we will assume a packet based IP network [19].

Delay / Latency

Let's start with the network metrics that can be technically measured. The most obvious one is the end to end packet delay when traversing a network. It is usually noted as round-trip time, measured in *seconds* or *milliseconds* (for clarity) and can be viewed as a double of the sum of then link latencies of links along the path of the packet plus the packet processing time at each node, considering the node/link capacity is *infinite*. In real networks, it is also affected by the node and link utilization [20]. It is usually considered that nodes and links capacity and utilization can be represented as M/M/1 queue, but recent studies on IP networks that delay is usually worse than the calculated by the Erlang C formula assuming the Poisson arrival process [21]. The flow and packet arrivals are anything but Poisson, on the other hand, and even exhibit the extreme correlation of self-similar processes [22] or having peak factors, that influence the queue waiting time[23].

For evaluation of a network, the average delay parameter is used. This can be considered as the mean of round-trip delays of all paths of the network [24], [25].

Jitter

By definition jitter is the deviation from true periodicity of a presumably periodic signal. In the context of packet based IP networks, jitter is the variation in latency as measured in the variability over time of the packet latency across a network. A network with constant latency has no variation (or jitter). Packet jitter is expressed as an average of the deviation from the network mean latency. However, for this use, the term is imprecise. The standards-based term is "packet delay variation" (PDV). However, for simplicity we can consider jitter is a change in end-to-end latency with respect to time in packet-based IP networks.

Packets arriving at a destination at a constant rate exhibit zero jitter, while packets from same flow, which are routed through different paths throughout the network, can experience very heavy jitter. [1]

Throughput

Throughput or **network throughput** is defined as the rate of *successful* message delivery over a communication channel. It is usually measured in bits per second (bit/s or bps), and sometimes in data packets per second (p/s or pps), depending of the purpose. For a network, the total throughput is the maximum rate of information transfer between each two nodes. For each discrete link the maximum throughput is limited by a huge number of factors. Some of these are described below:

Analog limitations - The maximum achievable throughput (the channel capacity) is affected by the bandwidth in hertz and SNR of the analogue physical medium. Despite the conceptual simplicity of digital information, all electrical signals traveling over wires are analogue. The analogue limitations of wires or wireless systems inevitably provide an upper bound on the amount of information that can be sent. The dominant equation here is the Shannon-Hartley theorem, and analogue limitations of this type can be understood as factors that affect either the analogue bandwidth of a signal or as factors that affect the signal to noise ratio [26].

Link latency and jitter - as of now, IP networks primarily carry TCP traffic [27], the link latency and the jitter limit the TCP throughput and it can be significantly affected due to reassembly of packets in a timely manner at the receiving end. So important to ensure that a particular TCP session is not drastically affected if packets for this session take two different paths through a network - note that these paths can conceivably have different latency even if both paths are marked as ECMP paths in the routing table. Thus, in most router implementations, packets that belong to a particular TCP session (i.e., going to a specific destination in terms of IP address of the end node) are routed on a specific shortest-path when multiple shortest paths are available. Packets for another TCP session, which may have the same pair of origin-destination routers as the first TCP session, can take a different shortest path than the first one (when multiple shortest paths are available). Thus the ECMP split may be decided based on per session, rather than on a packet-by-packet basis. This usually is achieved by hashing the packet headers and taking decisions based on the calculated hash. [20]

Packet loss due to **Network congestion**. Packets may be dropped in switches and routers when the packet queues are full due to congestion [28].

Packet loss due to **bit errors**. [26], [29]

Scheduling algorithms in routers and switches. If fair queuing is not provided, users that send large packets will get higher bandwidth. Some users may be prioritized in a weighted fair queuing (WFQ) algorithm if differentiated or guaranteed quality of service (QoS) is provided. [20], [30], [31]

So besides the purely physical constrains for the throughput, the link latency, the jitter and also the network utilization affect the overall throughput of the network.

Availability

By definition, availability is an expected average time in which a single device or the entire network operates properly. It usually measured as the probability that the system is working properly in the given moment of time. [32]

The availability of a network is a very wide term, which includes both topological aspects from the core of the network to the end users, and OSI model aspects from the physical layer to the application layer (as service availability). It is a KPI of networks and usually is directly connected to the QoE of services. As such, availability is in the ETSI QoS model and it is defined for each type of services [7].

As a rule of thumb, a network is reliable as its least reliable component, so measures to maximize the reliability of different components should be taken. Such measures include redundancy of links, nodes, ensuring there are several paths between each nodes, etc.

III. Graph properties

Graph theory has always been used for solving a number of problems in the field of Telecommunications. As each network can be represented as graph – either weighted or unweighted, either undirected, or directed – the graph theory can be used to find paths in the network, to calculate the shortest one or to search for critical edges (bridges) or critical nodes – called articulation points. Until recently most of these tasks were accomplished algorithmically. In the recent decades a new branch of graph theory has emerged – Spectral graph theory [33] [15]. Its foundations lie on the idea of representing the graph in matrix form – either adjacency matrix, or other type – and calculating the parameters of the graph based on the parameters of the matrices. Usually these parameters give a lot of information on the graph structure and its properties [34], both local and global.

Spectral graph theory is a mathematical theory in which linear algebra and graph theory meet. For any graph matrix M we can build a spectral graph theory in which graphs are studied by means of eigenvalues of the matrix M. Frequently used graph matrices are the adjacency matrix A, the Laplacian $L = D - A$ and the normalized Laplacian $\mathcal{L} = T^{-\frac{1}{2}} L T^{-\frac{1}{2}}$, where D is a diagonal matrix of vertex degrees. The Spectral graph theory includes all particular theories together with interaction tools.

Diameter

The diameter is not a specific spectral parameter of a graph, but is used a lot in spectral theory. The diameter of a graph is defined as the longest shortest-path between any two graph vertices (*i,j*), where $i \in V$ and $j \in V$ of a graph *G(V,E)*:

$$d_G = \max_{i,j} l_{i,j} \tag{1}$$

This implies the use of BFS algorithm to travel through the graph to find all paths [35].

It also can be defined as the maximum eccentricity among the vertices of *G*, as follows:

$$d_G = \max_{v \in V} \epsilon(v) \tag{2}$$

The diameter of graph is fundamental parameter of the networks, as it gives quick estimation of the size and worst case of paths through the network.

Average shortest-path length

The average shortest-path is also not a spectral specific parameter of graphs, but is very significant in the network graph analysis. It is defined as:

$$a_G = \sum_{s,t \in V} \frac{d(s,t)}{n(n-1)} \tag{3}$$

where V is the set of nodes in G, $d(s, t)$ is the shortest path from s to t, and n is the number of nodes in G.

The average shortest path length of a network is an important property, since it is directly correlated with how different parts of the network communicate, and exchange information. A small average distance is a prerequisite for improved synchronizability, efficient computation and signal propagation across the network [36], [37].

Spectrum of a graph

The spectrum of finite graph G_c is by definition the spectrum of the adjacency matrix *A*, its set of eigenvalues together with their multiplicities. The Laplace spectrum of finite graph G_c is the spectrum of the Laplace matrix *L* [33].

Since *A* is real and symmetric, all its eigenvalues are real. Also, for each eigenvalue λ_n, its algebraic multiplicity coincides with its geometric multiplicity. Since *A* has zero diagonal, its trace *tr(A)*, and hence the sum of the eigenvalues is zero.

Similarly, *L* is real and symmetric, so that the Laplace spectrum is real. Moreover, *L* is positive semi definite and singular, so we can denote the eigenvalues by:

$$\lambda_n \geq \cdots \geq \lambda_2 \geq \lambda_1 = 0 \tag{4}$$

Where for connected graph with single component is:

$$\lambda_n \geq \cdots \geq \lambda_2 > \lambda_1 = 0 \tag{5}$$

Actually the number of eigenvalues for which $\lambda_n = 0$, gives the number of connected components of a graph [38].

The sum of these eigenvalues is *tr(L),* which is twice the number of edges of G_c. Finally, also $\mathcal{L}$ has real spectrum and nonnegative eigenvalues (but not necessarily singular) and $tr(\mathcal{L})=tr(L)$.

From the spectrum of graph the most used thing is the Fiedler vector. According to [39], [40] this vector is the eigenvector associated with λ_2. This vector gives the spectral partitioning of the graph, from which we can derive the subcomponents by their relative connectivity. Also in [41] is introduced the symmetry ratio of a network, which metric is defined to be the ratio of the number of distinct eigenvalues of the network to the diameter. This metric is used to study the robustness of a network topology in the face of targeted attacks.

Algebraic connectivity

In [42] the algebraic connectivity $a(G_C)$ of a (connected) graph is defined as the second smallest eigenvalue (λ_2) of the Laplacian matrix of a graph with *n* vertices.

This parameter is used as a generalized measure of “how well is the graph connected”[43], [44]. It has values between *0* and *n* (a fully-connected graph C_n has *n*). This eigenvalue is greater than 0 if and only if G is a connected graph. This is a corollary to the fact that the number of times 0 appears as an eigenvalue in the eigenvector of the Laplacian is the number of connected components in the graph. Therefore, the farther λ_2 is from zero, the more difficult it is to separate a graph into independent components. However, the algebraic connectivity is equal to zero for all disconnected networks. Therefore, as soon as the connectedness is lost, due to failures for example, this measure becomes less useful by being too coarse.

For comparison of the connectivity of graphs with different number of nodes, it is useful to use the normalized Laplacian matrix of graphs and its spectrum [45], [15].

Node degree and number of edges

These are the most basic properties of any graph, which even define the graph itself. The node degree describes the number of neighbours a node has. The node degree distribution is the probability $P_r(k)$ that a randomly selected node has a given degree *k*. The number of links that on average connect to a node is called the average node degree. The average node degree can be easily obtained from the degree distribution through

$$E[D] = \sum_{k=1}^{D_{max}} k P_r(k) \tag{6}$$

where D_{max} is the maximum degree in a given graph.

For graph representing real communication networks, the node degree maps to the number of interfaces on the routers/switches, while the number of edges are the connections (both real and virtual, depending on the OSI layer on which the graph representation is made). The main goal of network optimization is keeping the number of edges small (keeping the graph sparse), while improving the overall network metrics. On the other hand the graph with the best possible network metrics is the full graph – C_n. So optimizing the number of links requires balancing the trade-off between high performance and redundancy vs. lower cost, power, and less inter-node links.

Network criticality

The term network criticality is introduced in [25]. It is a graph-theoretic metric that quantifies network robustness, and that was originally designed to capture the effect of environmental changes in core networks. It is defined as the average random-walk betweenness of a link (node) normalized by its weight. This quantity is independent of link (node) location and it is a decreasing and strictly convex function of link weights. Network criticality can be written in terms of the components of the undirected Moore-Penrose Laplacian matrix:

$$\hat{\tau} = \frac{2}{n-1} tr(L^{+}) \quad (7)$$

There is a useful interpretation of network criticality in terms of electrical circuits: network criticality is the unweighted average of the equivalent resistances [46]. Therefore optimizing criticality is equivalent to minimizing the average resistance or maximizing the average conductance of a network, which explains why network criticality can be considered also as a global robustness metric [47].

Effective Network resistance

Another good measure for network robustness is the effective resistance. The normalized total effective resistance is proportional to the inverse total effective resistance, which is defined as the sum of the pair wise effective resistances over all pairs of vertices [48]. The total effective resistance R^{tot} is the sum of the effective resistances over all pairs of vertices, where the effective resistance of the edges is defined as:

$$R_{ab} = \frac{v_a - v_b}{Y} \quad (8)$$

And the total effective resistance is:

$$R^{tot} = \sum_{i=1}^{n} \sum_{j=i+1}^{n} R_{ij} \quad (9)$$

In the literature the total effective resistance is also called Kirchhoff index. As a result of Klein and Randić work[19], it can be written as a function of the non-zero Laplacian (weighted) eigenvalues (for single component connected graphs):

$$R^{tot} = n \sum_{i=2}^{n} \frac{1}{\lambda_i} \quad (10)$$

For network robustness index the value of normalized effective resistance is used. The advantage of this is that values lie in the interval *[0;1]* and a large value indicates a robust network. It is zero for unconnected graphs and maximal (one) for complete graphs – similar to the algebraic connectivity:

$$R_{ab} = \frac{v_a - v_b}{Y} \quad (11)$$

IV. Correlation between the network metrics and graph properties

From the reviewed so far network metrics and graph properties, it is obvious that each one has its role in the network analysis. Moreover most these metrics and properties and bound to each other and there are correlations between them. Let's take as example the graph property algebraic connectivity – it is considered as a general indicator of how "well-connected" a graph is. Being "well-connected" implies good average path lengths as well as an abundance of loops to ensure good reliability and overall connectivity. Thus, graphs with high algebraic connectivity generally indicate efficient placement of links with many redundant paths between nodes, as well as good distribution of traffic (depending on the routing algorithm used). In figure 1 is shown the distribution of the average shortest path (Y-axis) and algebraic connectivity (X-axis) for 1000 random connected graphs – *G(n,p),* described with Erdős–Rényi model with *n>100* and *n<300*, and *0<p<1,* with beta distribution of random values of p with α*=1* and β*=50,* so it focuses more on sparse graphs to get better abstraction of networks [49]. The algebraic connectivity is normalized as per [50], so graphs with different number of nodes can be compared in terms of algebraic connectivity.

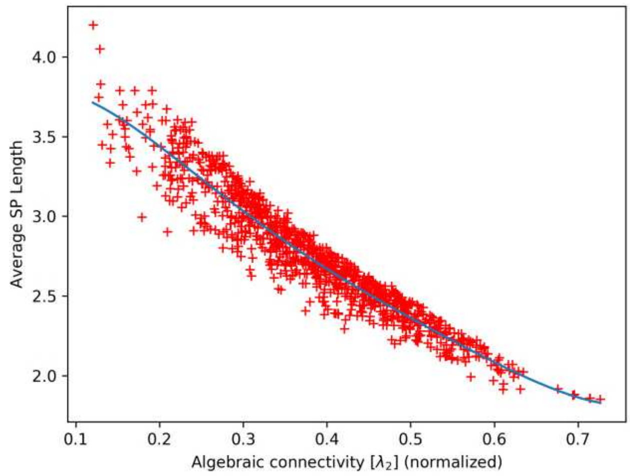

Fig.1. Average SP length vs. Algebraic connectivity for 1000 random graphs – G(n,p).

From the figure, it can be concluded that increase of the algebraic connectivity leads to shorten average shortest-path in a graph in almost linear fashion. The average shortest-path is in direct relation to the average delay of the network due to lower number of links and nodes that traffic pass through the network.

It is almost the same in regards to the diameter of graphs, as shown on figure 2 where the initial data are the same as above.

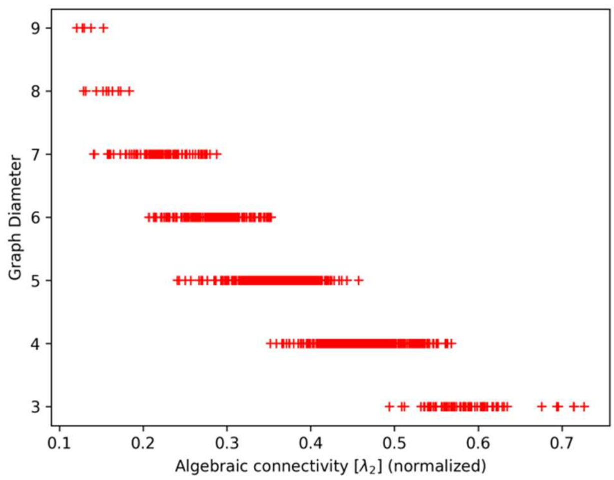

Fig.2. Diameter vs. Algebraic connectivity for 1000 random graphs – G(n,p).

Another interesting relation is between the average clustering coefficient of a graph and its algebraic connectivity, shown on figure 3. For unweighted graphs, the clustering of a node u is the fraction of possible triangles through that node that exist:

$$c_u = \frac{2T(u)}{deg(u)(deg(u) -)} \qquad (12)$$

where *T(u)* is the number of triangle through node *u* and *deg(u)* is the degree of *u*. And the average clustering coefficient of graph G is:

$$C = \frac{1}{n}\sum_{v \in G} c_v \qquad (13)$$

where *n* is the number of nodes in G.

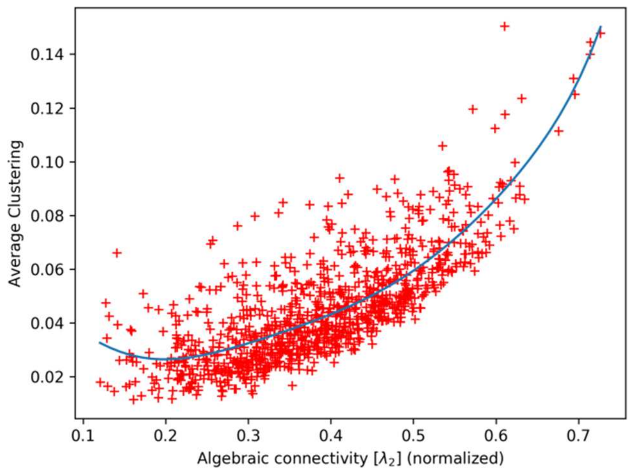

Fig.3. Average clustering coefficient vs. Algebraic connectivity for 1000 random graphs – G(n,p).

From the figure is seen that graphs with high algebraic connectivity also have higher average clustering coefficient, which means there are little weakly connected clusters and the graph edges are uniformly distributed between nodes. This leads to the exploration of the correlation between the clustering coefficient and the average path length, shown on figure 4.

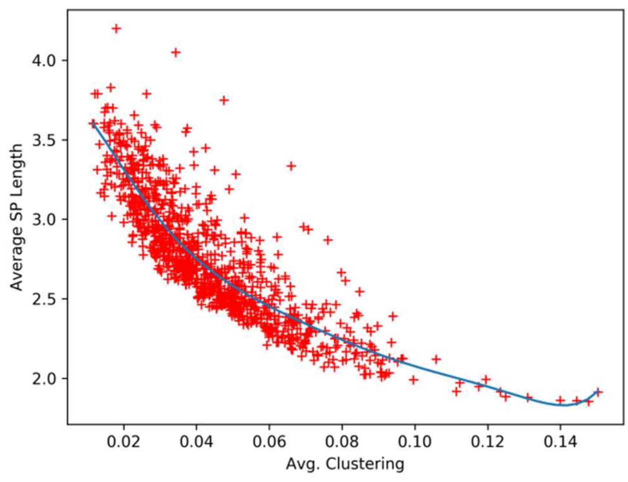

Fig.4. Average SP length vs. Average clustering coefficient for 1000 random graphs – G(n,p).

So if a network has both a high average clustering coefficient, and a small average distance, it is called a "small world" network [51]. This architecture have other desired properties, like enhanced signal propagation speed, synchronizability and computational power [37], [51]. As it turns out, the networks with the largest average clustering are "small world" networks, since they also have the smallest possible average distance [36]. This is also seen from figure 3, where the "best" algebraic connectivity leads to highest average clustering coefficient.

The last, but not least significant, is the network criticality property of graphs. It is bounded by the reciprocal of algebraic connectivity. More precisely:

$$\frac{2}{(n-1)\lambda_2} \leq \hat{\tau} \leq \frac{2}{\lambda_2} \qquad (14)$$

This means that increasing the connectivity of a network, decreases the upper bound of network criticality, which potentially means more robustness [52].

V. Conclusion and future works

In this paper are reviewed the basic network metrics, the graph properties and some correlations between each. As all these parameters are correlated between each other, the further works are on the more in-depth study of all the relationships between these parameters and finding optimal topologies by optimizing the graph spectral characteristics in given bounds. Also a future work is in the field of the solving the linear programming problem of achieving predefined spectral characteristics and creating a topology upon them.

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Received on: 20.04.2017