

Exploration of the possibilities of the variational approach for analysis of transient processes in electric circuits

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In this paper the possibilities for application of the variational approach for analysis of transient processes in linear and non-linear electric circuits are presented. It is based on two basic theorems, which were introduced several years ago for the needs of the variational analysis of electric circuits of all types, working in any type of regimes. A special method for analysis of electric circuits called method with the transferring coefficients was also developed and combined with the variational approach in order to avoid the usage of partial differential equations in the analyses of that type. The methodology, which allows the application of the variational approach for analysis of transient processes using the basic laws for the electric circuits, is revealed, too. A special methodology for optimization of the solutions by the help of the formulas of Newton-Cotes for numeric integration and the balance of powers is introduced, giving the chance to increase the precision of the solutions. Some examples with real electric circuits demonstrate the features and the scope of the variational method, too.

Изследване на възможностите на вариационния подход за анализ на преходни процеси в електрически вериги (Емил И. Панов, Мирослава Гр. Донева). В тази статия са представени възможностите за приложениена вариационния подход за анализ на преходни процеси в линейни и нелинейни електрически вериги. Той е базиран върху две основни теореми, които бяха въведени преди няколко години за целите на вариационния анализ на електрически вериги от всички видове и работещи при всякакви режими. Специален метод, наречен метод с предавателните коефициенти, също бе разработен и комбиниран с вариационния подход, за да се избегне работата с частни диференциални уравнения при анализите от този тип. Методиката, която позволява прилагането на вариационния подход за анализ на преходни процеси, използвайки основните закони на електротехниката, е също показана. Въведена е специална методика за оптимизиране на решенията с помощта на формулите на Нютон-Коутс за числено интегриране и баланса на мощностите, давайки възможност за увеличаване на точността на решенията. Някои примери с реални електрически вериги демонстрират също особеностите и обхвата на вариационния метод.

Introduction

The variational analysis of electric circuits is rarely used in electrical engineering. There are few examples for its application about some electrical processes and the corresponding powers, connected with them. In circuit theory the first systematical methodologies for variational analysis of electric circuits were introduced comparatively soon [1]-[11]. Because of the variety of different types of regimes, it is necessary these methodologies to be additionally developed and specified for the different cases of analysis of electric circuits. In the present paper a variational methodolo-

gy for analysis of transient processes in linear and non-linear electric circuits is presented.

Two dual theorems [5] can be used directly for that purpose, because they introduce the bases of the variational analysis of electric circuits. These theorems correspond to a similar theorem in electrostatics called “uniqueness theorem”, which is connected with the fact, that there is only one correct solution of each field task ([12], page 25). This theorem is a consequence of the principle of least action, which is valid for all aspects of physics ([13], from page 19-1 to page 19-14).

First theorem: For each EC, among all sets of currents, which formally satisfy Kirchhoff's current law (KCL) for the nodes of the circuit, there is only one set of currents, for which the instantaneous power of each current source has an extremum (a minimum or a maximum), if this power is not equal to zero. And this set of currents is the only one, which satisfies the equations using Ohm's and Kirchhoff's laws for the circuit.

Second theorem: For each EC, among all sets of voltages, which formally satisfy Kirchhoff's voltage law (KVL) for the loops of the circuit, there is only one set of voltages, for which the instantaneous power of each voltage source has an extremum (a minimum or a maximum), if this power is not equal to zero. And this set of voltages is the only one, which satisfies the equations using Ohm's and Kirchhoff's laws for the circuit.

In [5], [6], [7], [8], [9], [10], [11] some special techniques for analysis of electric circuits were developed on the base of the so-called transferring coefficients, which connect the explored quantities (voltages or currents) with the reference source and its quantity – the electromotive force (e.m.f.) or the electromotive current (e.m.c.). After the choice of the reference source and the introduction of the transferring coefficients, one equation is created by the help of the balance of the powers and after the exclusion from it of all coefficients except one, the resultant equation may be solved directly or it may be differentiated in order to find the extremum and the value of the basic transferring coefficient of the explored circuit. This approach can be combined with the nodal approach or the loop analysis, too. So, a big variety of methodologies was introduced for DC or AC analysis (the last type was connected with the phasor approach).

Analysis

Basic methodology for analysis of transient processes in electric circuits using variational approach.

The analysis of transient processes in the electric circuits by the help of the variational approach needs a little bit different procedure, because the basic

calculation technology is numeric one (especially for the analysis of non-linear electric circuits).

The methodology for numeric variational analysis of the transient processes in one electric circuit (for example with one reactive element) has the following steps:

1) A reference source (of e.m.c. with a current $j_1(t)$ or e.m.f. with a voltage $e_1(t)$) is selected in the circuit being studied. A pair of terminals (a) and (b), separating the circuit into a reference source and a resistive part, is introduced, while all other sources (of e.m.f. or e.m.c.) are considered to be resistors according to the compensation theorem with positive or negative resistances $R_{eq}(t)$ or $R_{js}(t)$.

2) A number of m transferring coefficients k_1, k_2, \dots, k_m are introduced in relation to the currents flowing through the elements (the voltage drops upon them) having in mind KCL or KVL, where:

$$i_1(t) = k_1(t) \cdot j_1(t),$$

$$i_2(t) = k_2(t) \cdot j_1(t), \dots, i_m(t) = k_m(t) \cdot j_1(t),$$

or

$$(u_1(t) = k_1(t) \cdot e_1(t), u_2(t) = k_2(t) \cdot e_1(t), \dots,$$

$$u_m(t) = k_m(t) \cdot e_1(t)).$$

3) A system of $(m-1)$ equations is created by the help of KVL (or KCL) and the system is solved taking into account one of the coefficients to be a parameter, for example $k_1(t)$, i.e.

$$k_2(t) = f_2(k_1(t)), k_3(t) = f_3(k_1(t)), \dots,$$

$$k_m(t) = f_m(k_1(t)),$$

where $k_1(t)$ is the transferring coefficient of the reactive element.

4) One differential equation of the following normalized form is created for the explored circuit:

$$(1) \quad \frac{dk_1(t)}{dt} = f[k_1(t); R_1; \dots; R_q; C \text{ (or } L)],$$

which will be the predictor for the calculating process. Then, several initial values can be calculated by the help of the method Runge–Kutta–4 for the first p steps with a step size h in the time interval $(t_0; t_p)$.

5) Another equation is created on the base of the balance of powers for the instantaneous powers of the elements of the explored circuit:

$$(2) \quad \int_{t_0}^{t_p} \left\{ P_{source}[k_1(t)] - \sum_{s=1}^q P_{R_s}[k_1(t)] \right\} dt = ,$$

$$= \int_{t_0}^{t_p} P_{R.E.}[k_1(t)] dt = W_{R.E.}[k_1(t_p)] - W_{R.E.}[k_1(t_0)]$$

where $P_{R.E.}$ and $W_{R.E.}$ are the instantaneous power of the reactive element and the corresponding accumulated energy.

Equation (2) will play the role of a corrector of the calculation process.

6) Then, the first optimization may be started in order to precise the first p values, received by equation (1), using the corrector – equation (2). The procedure is maintained by the usage of iteration procedure and a numeric integration formula of Newton – Cotes of rank higher than 4 (for example it may be the 6th order integration formula).

$$(3) \quad \int_{t_0}^{t_p} \left\{ P_{source}[\alpha.k_1(t)] - \sum_{s=1}^q P_{R_s}[\alpha.k_1(t)] \right\} dt = .$$

$$= W_{R.E.}[\alpha.k_1(t_p)] - W_{R.E.}[\alpha.k_1(t_0)]$$

The calculation procedure starts with an initial value for the correction coefficient $\alpha_{(0)} = 1$. After the calculation process is over, the first $p+1$ values of $k_1(t)$ are improved, i.e. we receive the precise values $k_1(t_0)$; $k_1(t_1)$; ...; $k_1(t_p)$ for the next calculation process.

7) By the help of the predictor – education (1) we can get the next value $k_1(t_{p+1})$, which we can improve by the corrector – equation (2):

$$(4) \quad \int_{t_0}^{t_p} \left\{ P_{source}[k_1(t)] - \sum_{s=1}^q P_{R_s}[k_1(t)] \right\} dt = ,$$

$$= W_{R.E.}[k_1(t_{p+1})] - W_{R.E.}[k_1(t_1)]$$

using an iteration procedure and a high order integration formula of Newton – Cotes (for example it may be 6th order integration procedure). This is the second optimization, which can be repeated over and over again till the end of the transient process.

Numeric examples for variational analysis of transient processes in electric circuits by the help of transient coefficients

Example 1: A linear electric circuit is presented in Fig.1 with the following parameters: $R = R_1 = R_2 = 50 \Omega$; $C = 50 \mu F$; $e = 1 V$. Find the voltage drop across the capacitor $u_C(t)$ by the variational approach.

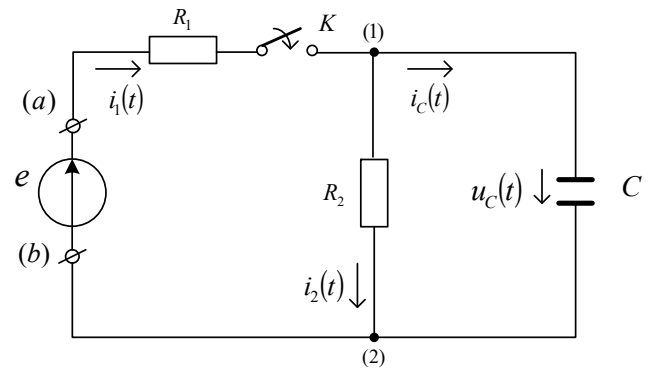


Fig.1. A linear electric circuit.

Solution:

The voltage across the capacitor can be expressed as: $u_c(t) = k(t).e$. Then, the currents $i_1(t)$ and $i_2(t)$ will be: $i_1(t) = \frac{[1-k(t)]e}{R_1}$; $i_2(t) = \frac{k(t).e}{R_2}$. The predictor of the calculation process will be:

$$(5) \quad \frac{dk(t)}{dt} = \frac{1-2k(t)}{RC} ,$$

which has an exact classical solution

$$(6) \quad k(t) = 0,5 \left(1 - e^{-\frac{2t}{RC}} \right) ,$$

i.e. $u_c(t) = k(t).e = k(t).1 = 0,5 \left(1 - e^{-\frac{2t}{RC}} \right) .$

The corrector for the calculation process will have the following form:

$$\int_{t_0}^{t_6} [p_e(t) - p_{R_1}(t) - p_{R_2}(t)] dt = \int_{t_0}^{t_6} p_c(t) dt, \text{ or}$$

$$\int_{t_0}^{t_6} [e.i_1(t) - R_1.i_1^2(t) - R_2.i_2^2(t)] dt =$$

$$= \int_{t_0}^{t_6} C \frac{du_c(t)}{dt} .u_c(t) dt = \int_{t_0}^{t_6} C u_c(t) du_c(t),$$

or

$$\int_{t_0}^{t_6} \left\{ e \frac{[1-k(t)]e}{R_1} - R_1 \frac{[1-k(t)]^2 . e^2}{R_1^2} - R_2 \frac{k(t)^2 . e^2}{R_2^2} \right\} dt =$$

$$= C \int_{t_0}^{t_6} k(t) . e . d[k(t) e] = \frac{C e^2}{2} \int_{k(t_0)}^{k(t_6)} k(t) . dk(t) . \text{ I.e.}$$

$$\frac{e^2}{R} \int_{t_0}^{t_6} \{ k(t) [1 - 2k(t)] \} dt = C e^2 [k^2(t_6) - k^2(t_0)] \text{ or}$$

$$\frac{2}{RC} \int_{t_0}^{t_6} \{ k(t) [1 - 2k(t)] \} dt + k^2(t_0) = k^2(t_6) .$$

At the end we have:

$$(7) \quad k(t_6) = \frac{2}{RC} \int_{t_0}^{t_6} \sqrt{\{ k(t) [1 - 2k(t)] \} dt + k^2(t_0)} .$$

Here, the numeric integration can be fulfilled by the 6th order Newton-Cotes formula:

$$(8) \quad \int_{t_0}^{t_6} f(t) dt =$$

$$= \frac{(t_6 - t_0)}{840} \left[41.f(t_0) + 216.f(t_1) + 27.f(t_2) + \right.$$

$$\left. + 272.f(t_3) + 27.f(t_4) + \right.$$

$$\left. + 216.f(t_5) + 41.f(t_6) \right]$$

The first 6 values for $k(t)$, i.e. $k(t_1), \dots, k(t_6)$ can be received from the corrector - equation (5) by the

help of the method Runge-Kutta-4 and then, they can be put under the first optimization iterative procedure:

$$\alpha_{(r+1)} . k(t_6) =$$

$$(9) \quad = \sqrt{\frac{2}{RC} \int_{t_0}^{t_6} \{ \alpha_{(r)} . k(t) [1 - 2\alpha_{(r)} . k(t)] \} dt + [\alpha_{(k)} . k(t_0)]^2},$$

where $\alpha_{(0)} = 1$ is the initial value for the optimization procedure. After its completion, all the first six values of $k(t)$ can be improved by the received correction coefficient:

$$k_{cor}(t_1) = \alpha_{(r+1)} . k(t_1); \quad k_{cor}(t_2) = \alpha_{(r+1)} . k(t_2); \quad \dots ;$$

$$k_{cor}(t_6) = \alpha_{(r+1)} . k(t_6) .$$

After we have already improved the six initial values of the transferring coefficient $k(t)$, we can improve the seventh one - $k(t_7)$, too. For that purpose we can use equation (7), which is our corrector, and it will have the following form:

$$(10) \quad k(t_7) = \frac{2}{RC} \int_{t_1}^{t_7} \sqrt{\{ k(t) [1 - 2k(t)] \} dt + k^2(t_1)} .$$

For that purpose we use the predicted value $k(t_7)$, received from the predictor (equation (5)) and we may better it by iterative procedure using equations (8) and (10). This is the second optimization procedure for the values $k(t_7), k(t_8), k(t_9), \dots$ till the end of the transient process we explore.

In Fig.2 the relative error d is presented for two calculation procedures – the first by the help of the method Runge-Kutta-4 and the second for the variational approach, where:

$$d = \left| \frac{k_{exact}(t) - k_{calc}(t)}{k_{exact}(t)} \right| . 100\% .$$

Here, the step size is $h = \frac{1}{6} ms$; the first optimization is completed at $t = 1 ms$; the second optimization starts right away after the first optimization.

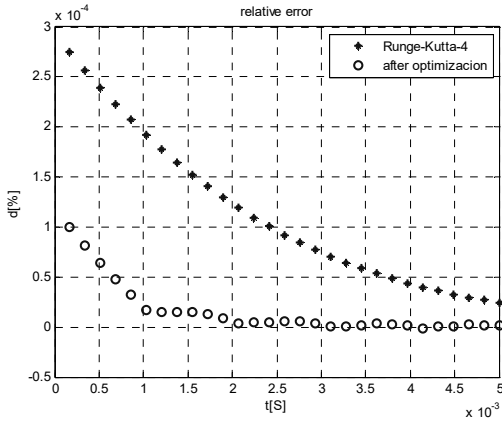


Fig.2. The relative error versus time.

In Fig.3 the absolute error $D = |k_{exact}(t) - k_{calc}(t)|$ is presented for both methods of calculation.

In the last two figures it is well seen, that the variational approach can improve the results of one of the most often used numeric methods for solution of ordinary differential equations - the method Runge-Kutta-4.

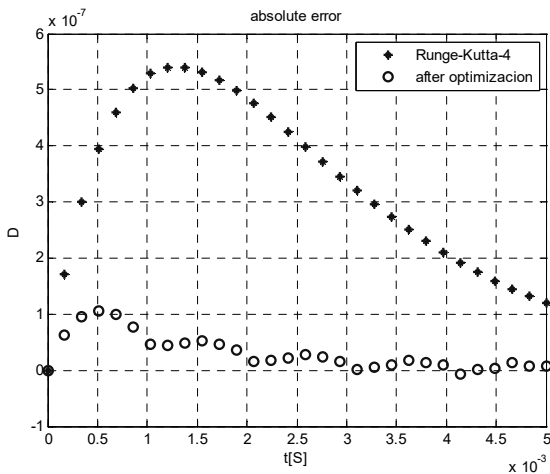
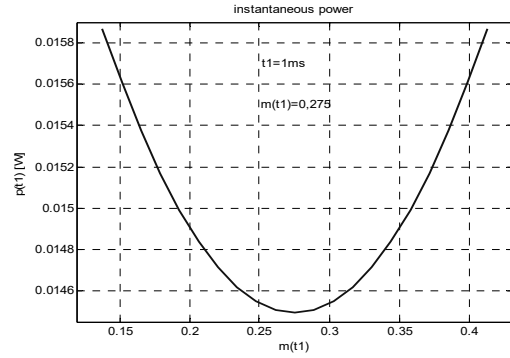
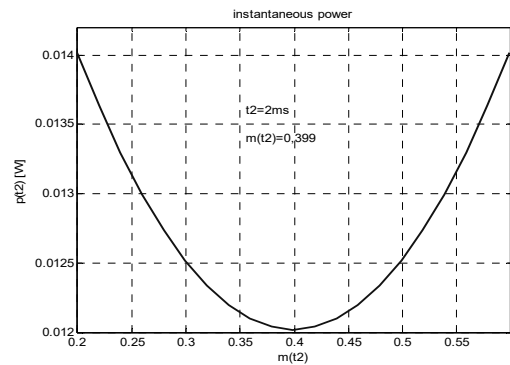


Fig.3. The absolute error versus time.

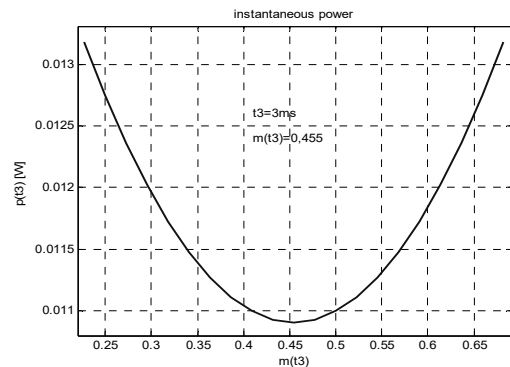
In Fig.4 (a), (b), (c), (d) and (e) the extremum of the function $p_{source}[k(t)]$ is presented for the moments $t_1 = 1\text{ ms}$; $t_2 = 2\text{ ms}$; $t_3 = 3\text{ ms}$; $t_4 = 4\text{ ms}$ and $t_5 = 5\text{ ms}$. It is well seen that for the calculated values of the transferring coefficient in these five moments, the voltage source spends minimum power in order to support the transient process of the electric circuit.



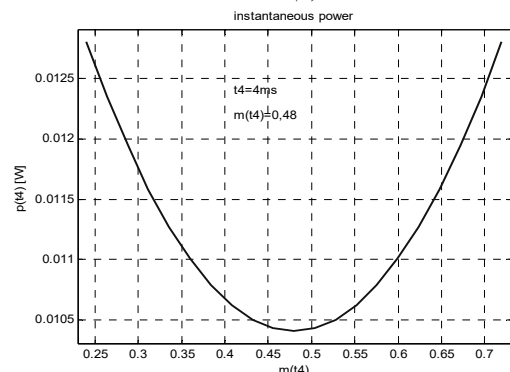
(a)



(b)



(c)



(d)

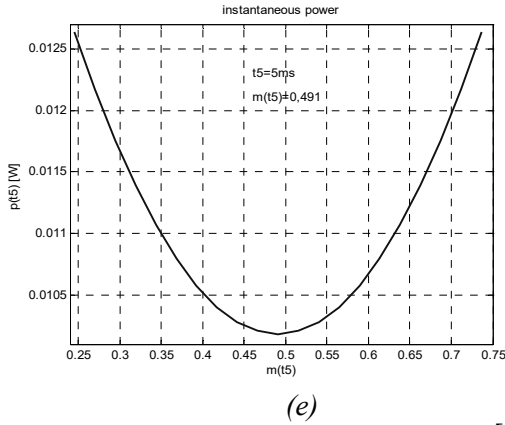


Fig.4. The extrema of the function $p_{source}[k(t)]$.

The solution of that task is more complex compared with the solutions made by other classical methods. Here, the main goal is not to find any voltage or current in the quickest way with less calculations, etc. The transient processes in linear or non-linear circuits are subordinated to the principle of least action at each moment of time. And that example is presented in order to feel directly that hidden fact.

The next example is a step towards the same direction.

Example 2: A non-linear electric circuit is presented in Fig.5, where: $R_1=120 \Omega$; $R_2=5 \Omega$; $R_3=0,2 \Omega$; $e=250 V$; $i_L(t)=0,5 \Psi_L^2(t)$. Find the current flowing through the coil $i_L(t)$ by the variational approach.

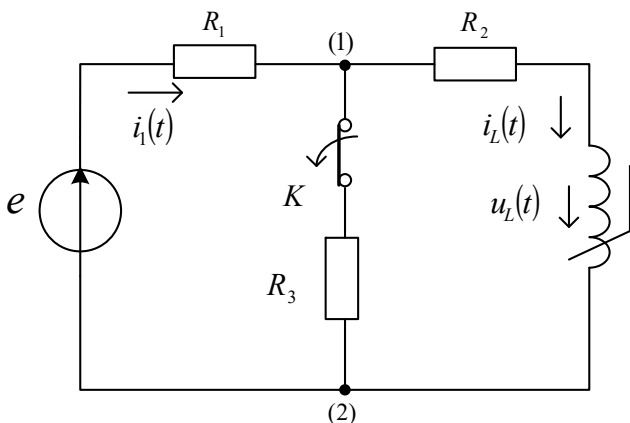


Fig.5. A non-linear electric circuit.

Solution:

In Fig.6 the equivalent electric circuit with a non-ideal current source is presented. The current flowing through the non-linear coil can be expressed as:

$$i_L(t) = k(t) j, \text{ where } j = \frac{e}{R_1} = \frac{25}{12} A.$$

Then:

$$\begin{aligned} u_L(t) &= \frac{d\Psi_L(t)}{dt} = \frac{d}{dt} [\sqrt{2} \cdot i_L(t)] = \\ &= \frac{d}{dt} [\sqrt{2} \cdot k(t) \cdot j] = \frac{dk(t)}{dt} \cdot \sqrt{\frac{j}{2 \cdot k(t)}} \end{aligned}$$

$$\text{and } i_1(t) = j \cdot [1 - k(t)].$$

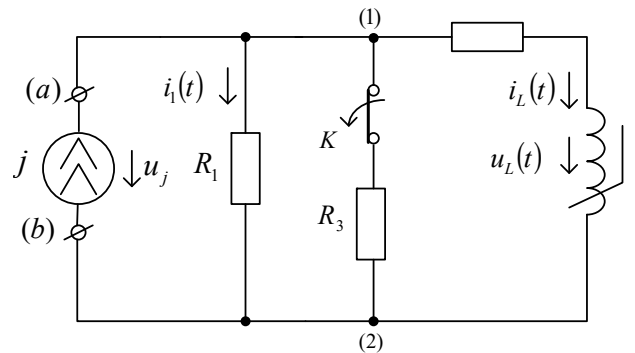


Fig.6. The equivalent electric circuit.

Using KCL and KVL we can express the voltage drop $u_j(t)$, i.e. $u_j(t) = R_1 \cdot i_1(t) = R_1 \cdot j \cdot [1 - k(t)]$ and we can also find the connection among $u_j(t)$, $i_L(t)$ and $u_L(t)$: $u_j(t) = R_2 i_L(t) + u_L(t)$. From here, we receive the predictor equation:

$$(11) \quad \frac{dk(t)}{dt} = \sqrt{2 k(t) \cdot j} \cdot [R_1 - (R_1 + R_2) \cdot k(t)].$$

The exact classical solution of that equation is:

$$(12) \quad k(t) = \frac{12}{50} \left(\frac{3 - 2e^{-250t}}{1,5 + e^{-250t}} \right)^2$$

$$\text{and } i_L(t) = k(t) \cdot j = 0,5 \left(\frac{3 - 2e^{-250t}}{1,5 + e^{-250t}} \right)^2 A.$$

The corrector for the calculation process will have the following form:

$$\int_{t_0}^{t_6} [p_j(t) - p_{R_1}(t) - p_{R_2}(t)] dt = \int_{t_0}^{t_6} p_L(t) dt,$$

where

$$\int_{t_0}^{t_6} [u_j(t) \cdot j - R_1 \cdot i_1^2(t) - R_2 \cdot i_L^2(t)] dt =$$

$$= \int_{t_0}^{t_6} u_L(t) \cdot i_L(t) dt$$

or

$$\int_{t_0}^{t_6} \left\{ R_1 \cdot j^2 \cdot [1 - k(t)] - R_1 \cdot j^2 \cdot [1 - k(t)]^2 - R_2 \cdot j^2 \cdot k^2(t) \right\} dt =$$

$$= \int_{t_0}^{t_6} \left[\frac{dk(t)}{dt} \sqrt{\frac{j}{2k(t)}} \right] \cdot k(t) \cdot j dt,$$

or

$$j^2 \cdot \int_{t_0}^{t_6} [R_1 \cdot k(t) - (R_1 + R_2) \cdot k^2(t)] dt =$$

$$= \frac{\sqrt{2}}{3} j^{3/2} [k^{3/2}(t_6) - k^{3/2}(t_0)],$$

or

$$k(t_6) =$$

$$(13) \quad = \left\{ 2,5\sqrt{1,5} \int_{t_0}^{t_6} [R_1 \cdot k(t) - (R_1 + R_2) \cdot k^2(t)] dt + \right. \\ \left. + k^{3/2}(t_0) \right\}^{2/3}.$$

Here, the numeric integration can be fulfilled by the 6th order Newton-Cotes formula (8).

The first 6 values for $k(t)$, i. e. $k(t_1); \dots; k(t_6)$ can be received from the corrector - equation (13) by the help of the method Runge-Kutta-4 and then, they can be put under the first optimization iterative procedure:

$$\alpha_{(r+1)} \cdot k(t_6) =$$

$$(14) \quad = \left\{ 2,5\sqrt{1,5} \int_{t_0}^{t_6} \left\{ R_1 \cdot \alpha_{(r)} \cdot k(t) - \right. \right. \\ \left. \left. - (R_1 + R_2) \cdot [\alpha_{(r)} \cdot k(t)]^2 \right\} dt + \right. \\ \left. + [\alpha_{(r)} \cdot k(t_0)]^{3/2} \right\}^{2/3},$$

where $\alpha_{(0)} = 1$ is the initial value for the optimization procedure. After its completion, all the first six values of $k(t)$ can be corrected by the received correction coefficient:

$$k_{cor}(t_1) = \alpha_{(r+1)} \cdot k(t_1); \quad k_{cor}(t_2) = \alpha_{(r+1)} \cdot k(t_2); \quad \dots; \\ k_{cor}(t_6) = \alpha_{(r+1)} \cdot k(t_6).$$

After we have already improved the six initial values of the transferring coefficient $k(t)$, we can improve the seventh one - $k(t_7)$, too. For that purpose we can use equation (13), which is our corrector, and it will have the following form:

$$k(t_7) =$$

$$(15) \quad = \left\{ 2,5\sqrt{1,5} \int_{t_1}^{t_7} [R_1 \cdot k(t) - (R_1 + R_2) \cdot k^2(t)] dt + \right. \\ \left. + k^{3/2}(t_1) \right\}^{2/3}.$$

For that purpose we use the predicted value $k(t_7)$, received from the predictor (equation (11)) and we may improve it by iterative procedure using equations (8) and (15). This is the second optimization procedure for the values $k(t_7); k(t_8); k(t_9); \dots$, till the end of the transient process we explore.

In Fig.7 the relative error d is presented for two calculation procedures – one for the Runge–Kutta-4 method and another for the optimized numeric solution of the variational approach. Here, the step size is $h = \frac{1}{3} ms$; the first optimization is completed at $t = 2ms$; the second optimization starts right away after the first optimization.

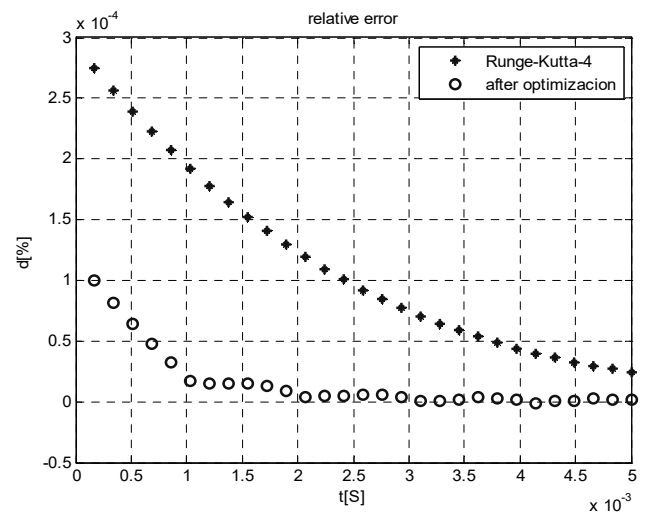


Fig.7. The relative error versus time.

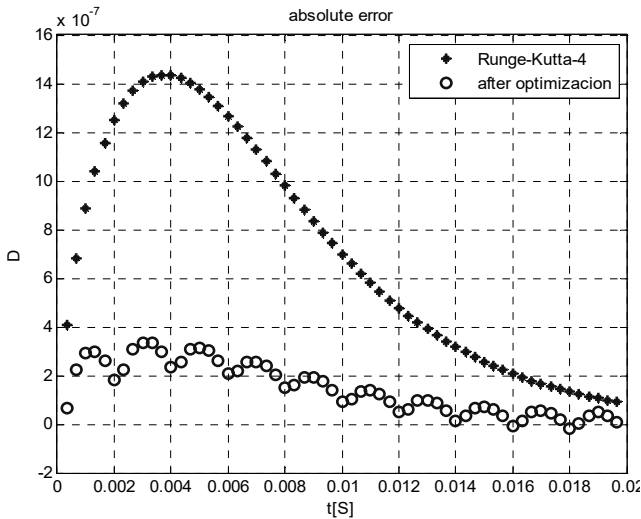


Fig.8. The absolute error versus time.

In Fig.8 the absolute error D is presented for both methods of calculation.

In the last two figures it is well seen, that the variational approach can improve the solution of one of the most often used numeric methods for solution of ordinary differential equations - the Runge-Kutta-4 method.

The truncation error of the method Runge-Kutta-4 is $\varepsilon_{T.R.K.} = \frac{h^5}{5!} f^{(V)}(\xi)$, where ξ is some point within the time interval of the last step h of the calculation process. The sixth order integration formula of Newton-Cotes has an error:

$$\varepsilon_{T.N.C.} = -\frac{h^7}{1400} [10 \cdot f^{(VI)}(\xi) + 9h^2 \cdot f^{(VIII)}(\eta)],$$

where ξ and η are some points within the integration interval $[t_k; t_{k+6}]$. So, for $h \ll 1$ it is clear, that $\varepsilon_{T.R.K.} \gg \varepsilon_{T.N.C.}$, which allows the Newton-Cotes formulas of higher order to be in the base of the corrector integration equations.

Conclusions

The proposed variational approach is more complex compared with the classic solutions. It is better to use it for analysis of transient processes in non-linear electric circuits, especially in cases when there are no exact classic solutions. The optimization procedures implemented in the variational method

allow to improve the numeric solutions and to increase the accuracy of the final results.

The proposed methodology can be ever introduced successfully, because we can always choose a corrector of higher order, compared with the order of the predictor, having in mind that the integration equations of Newton-Cotes form an infinite family of high-precision formulas. And the graphs in Fig.2, Fig.3, Fig.7 and Fig.8 illustrate in the best way that fact.

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